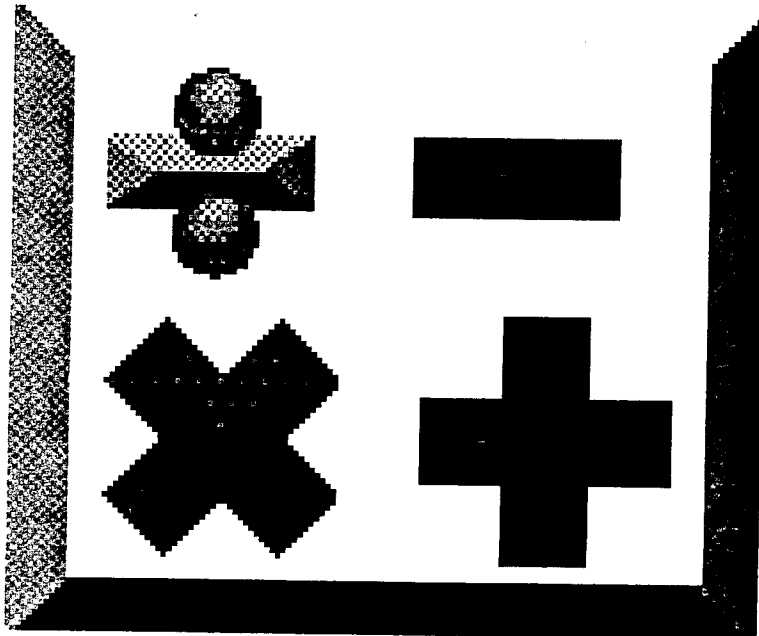




UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2010



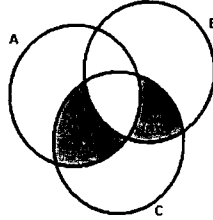
**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $9 + 1 \times 8 - 2 \div (7 - 3) \times (6 + 4) \div 5$

- (A) 1.875 (B) 7.5 (C) 16 (D) 30 (E) 39

2. Let $A = \{0, 2, 1, 3, 4, 7\}$, $B = \{0, 1, 2, 3, 5, 6, 8\}$, and $C = \{0, 1, 3, 5, 6, 7, 9\}$. How many elements are in the shaded areas of the Venn diagram shown?



- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3. On a map legend, $\frac{3}{4}$ inch represents 15 miles. The distance on the map from Ft. Stockton to Van Horn is 6 inches. How far is that in miles?

- (A) 120 (B) 105.75 (C) 90 (D) 75.25 (E) 67.5

4. Dr. Saul Tee had a bucket that contained 4 ounces of salt and 6 ounces of water. If he poured 70 ounces of water into the bucket, what percent of the mixture would be salt?

- (A) $1\frac{1}{7}\%$ (B) $3\frac{3}{7}\%$ (C) 5% (D) $5\frac{2}{3}\%$ (E) 7%

5. Noah Dough has a bag of nickels and dimes. He has a total of 42 coins. Les Cash has 8 dimes and 6 nickels in his pocket. The ratio of nickels to dimes is the same for both Les and Noah. How much more money does Noah have than Les?

- (A) \$3.30 (B) \$ 3.00 (C) \$2.20 (D) \$ 2.00 (E) \$ 1.10

6. Find an equation of a line through point $(2, -1)$ that is parallel to a line that contains point $(1, -2)$ and point $(2, 1)$.

- (A) $x - 3y = -1$ (B) $y = 3x - 7$ (C) $x + 3y = 1$ (D) $y = 3x + 5$ (E) $x - 2y = -3$

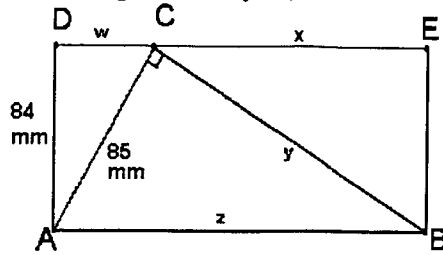
7. Two coplanar angles that have a common vertex, a common side, and have no common interior points are called _____ angles.

- (A) alternate (B) vertical (C) corresponding (D) adjacent (E) dihedral

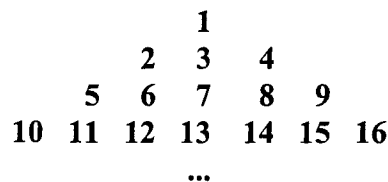
8. $\angle A$ and $\angle B$ are complementary angles. $\angle B$ and $\angle C$ are supplementary angles. Find $m\angle C$ if $m\angle A = 4x - 5$ and $m\angle B = 3x + 4$.

- (A) 137° (B) 133° (C) 127° (D) 123° (E) 117°

9. $\triangle ABC$ is drawn in the rectangle. Find y . (nearest whole number)



- (A) 536 mm (B) 549 mm (C) 562 mm (D) 583 mm (E) 598 mm
10. The *Local Snail* leaves the train station and runs at an average rate of 35 mph. An hour and a half later, the *Local Express* leaves the station and runs at an average rate of 56 mph on a parallel track. How long will it take the *Express* to overtake the *Snail*?
- (A) 4 hrs (B) 3.5 hrs (C) 2.5 hrs (D) 2 hrs (E) 1.6 hrs
11. Let $f(x) = 2x - 3$ and $g(x) = 3x + 1$ and $h(x) = 4 - x$. Find $g(f(h(5x)))$.
- (A) $30x - 4$ (B) $14 - 30x$ (C) $30x - 6$ (D) $23 - 30x$ (E) $16 - 30x$
12. If $y^2 = -3 - 4i$ and $y^3 = 11 - 2i$ where $y = a + bi$ then $a + b$ equals:
- (A) 4 (B) 3 (C) 1 (D) 0 (E) -1
13. Determine the range of $f(x) = 5\cos(4x - 3) - 2$.
- (A) $[-5, 5]$ (B) $[-2, 5]$ (C) $[-7, 3]$ (D) $[2, -5]$ (E) $[7, -3]$
14. How many elements are in $\{x \mid \sin^2(x) + \sin(x) - 6 = 0, x \in [0, 2\pi)\}$?
- (A) 4 (B) 3 (C) 2 (D) 1 (E) 0
15. The Red Baron leaves Snoopy airfield flying at a ground speed of 160 mph for 1 hour and 45 minutes on a bearing of 30° . Then he changes course and flies at a ground speed of 185 mph for 1 hour on a bearing of 130° . How far is the Red Baron from the airfield at this time? (nearest mile)
- (A) 278 mi (B) 308 mi (C) 124 mi (D) 345 mi (E) 103 mi
16. If the set of numbers $\{1, 2, 3, 4, 5, \dots\}$ continue in the triangular pattern shown below, then the median of the 9th row would be?



- (A) 56 (B) 63 (C) 71 (D) 73 (E) 76

17. In the binomial expansion of $(2x - y)^4$, the sum of the coefficients of the terms x^3y and xy^3 is:

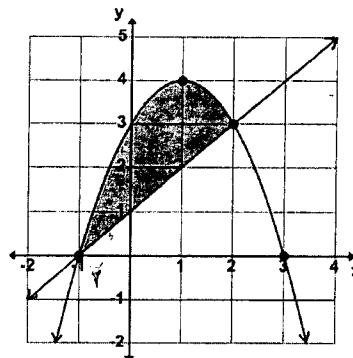
- (A) -24 (B) 40 (C) -10 (D) 24 (E) -40

18. Find the determinant of the 3×3 matrix.

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

- (A) 6 (B) 5 (C) 2 (D) -8 (E) -10

19. Find the area of the shaded region in square units.



- (A) 5 (B) $4\frac{3}{4}$ (C) $4\frac{1}{2}$ (D) $4\frac{1}{4}$ (E) $3\frac{1}{2}$

20. The directrix of the conic given by the equation $9x^2 - 16y^2 = 144$ is:

- (A) $x = 3.2$ (B) $y = 1.8$ (C) $x = 1\frac{1}{3}$ (D) $y = 1\frac{7}{9}$ (E) $x = 0.3125$

21. The point $(-1, -4)$ is rotated $\frac{17\pi}{6}$ radians counterclockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

- (A) $(-3.0, 2.9)$ (B) $(2.9, 3.8)$ (C) $(3.8, -1.6)$ (D) $(2.9, 3.0)$ (E) $(-1.6, -3.8)$

22. Find the angle of rotation, θ (nearest degree), where $0^\circ < \theta < 90^\circ$, such that the conic $4x^2 - 6xy - 9y^2 = 36$ contains no xy term in its equation.

- (A) 78° (B) 71° (C) 61° (D) 22° (E) 12°

23. There are 5 boys and 8 girls in the senior class at Millersview High School. Principal White wants an advisory committee made up of 4 seniors. He wants at least 1 girl and at least 1 boy. How many committees can be formed from the senior class?

- (A) 275 (B) 280 (C) 440 (D) 640 (E) 714

24. Mr. Lou Kuss bought a pair of blank dice. He wrote the numbers 2, 1, 3, 4, 7, and 11 on each die such that each face had only one number on it. He rolled the dice and added the two top faces. What are the odds that the sum was a prime number?

- (A) 5 to 7 (B) 7 to 23 (C) 3 to 10 (D) 7 to 29 (E) 13 to 23

25. Find the tens digit of $(2011)^{2010}$.

- (A) 0 (B) 1 (C) 4 (D) 8 (E) 9

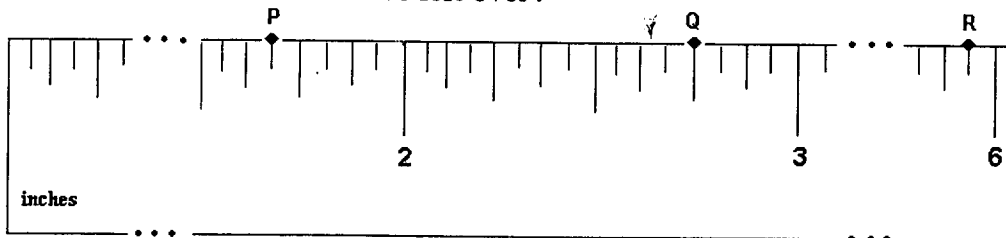
26. Yaps, Yeps, and Yips are three types of precious stones. Six Yaps has the same value as eight Yips. Three Yeps has the same value as five Yips. Rank the three stones in order of their value from highest to lowest.

- (A) Yap, Yep, Yip (B) Yip, Yap, Yep (C) Yep, Yip, Yap (D) Yep, Yap, Yip (E) Yap, Yip, Yep

27. One trillion minus two billion plus three million minus four thousand plus five hundred minus sixty plus seven equals K. How many digits in the number K only appear once?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

28. Pinkie Riban bought a foot of ribbon to use on her class project. She needs three lengths of ribbon. One piece has a length of P, a second piece has length Q, and a third piece has a length of R. How much ribbon will she have left over?



- (A) $\frac{3}{8}$ " (B) $1\frac{5}{8}$ " (C) $1\frac{3}{4}$ " (D) $1\frac{7}{8}$ " (E) $2\frac{1}{8}$ "

29. $44_5 + 333_4 + 2222_3 + 11111_2 = \underline{\hspace{2cm}}_{10}$

- (A) 198 (B) 196 (C) 137 (D) 110 (E) 99

30. Five car washers can wash 8 cars in 3 hours. How long will it take 4 car washers to wash 6 cars if the length of time it takes the washers to wash the cars varies directly as the number of cars and inversely as the number of washers? (nearest minute)

- (A) 1 hr 17 min (B) 5 hrs (C) 2 hrs 20 min (D) 5 hrs 7 min (E) 2 hrs 49 min

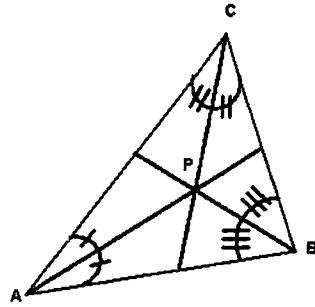
31. Find $f(-3) + f(-2) + f(-1)$ if $f(x) = \begin{cases} x-1 & \text{if } |x-1| > 3 \\ x & \text{if } |x-1| = 3 \\ x+1 & \text{if } |x-1| < 3 \end{cases}$

- (A) 6 (B) 3 (C) -5 (D) -6 (E) -9

32. Points P and Q lie on the x-y plane. Point P(3,0) is reflected across the line $y = -x$ to P' . Point Q(0, -2) is rotated 90° counterclockwise to Q' . Find the shortest distance between P' and Q' . (nearest tenth of a unit)

- (A) 2.2 (B) 2.4 (C) 3.1 (D) 3.3 (E) 3.6

33. Point P is the _____ of $\triangle ABC$ shown below.



- (A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

34. Ima Hogg has \$240.00 in her piggy jar. She decides to spend $\frac{1}{12}$ of the money remaining in the jar on the first day of the month, starting on June 1. At this rate, how much money will she have spent by May 30 the next year? (nearest cent)

- (A) \$20.00 (B) \$77.44 (C) \$147.84 (D) \$155.52 (E) \$162.56

35. If $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_n = a_{n-2} - a_{n-1} + a_{n-3}$, where $n \geq 4$, then a_8 equals:

- (A) -8 (B) -5 (C) -2 (D) 6 (E) 11

36. Find $\sin\left(\frac{\pi}{6}\right) \times \cos\left(\frac{\pi}{3}\right) \times \tan\left(\frac{\pi}{4}\right) \times \csc\left(\frac{\pi}{2}\right) \times \sec\left(\frac{2\pi}{3}\right) \times \cot\left(\frac{3\pi}{4}\right)$.

- (A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

37. A forest ranger marks a point x feet from the base of a cliff and a second point 500 feet farther from the first point. The angles of elevation from the points to the top of the cliff are 70° and 59° , respectively. Find the height of the cliff. (nearest foot)

- (A) 2111 ft (B) 3404 ft (C) 4880 ft (D) 4221 ft (E) 2571 ft

38. Find the harmonic mean of the real roots of $x^3 - 7.5x^2 + 15.5x - 6 = 0$.

- (A) $2\frac{9}{10}$ (B) $2\frac{1}{2}$ (C) $1\frac{15}{16}$ (D) $1\frac{4}{5}$ (E) $1\frac{5}{31}$

39. Let $f(x) = \frac{x^3 - 2x^2 - 8x}{x^2 - 9}$ and $s(x)$ be the slant asymptote of f . Find the value of $s(2)$.

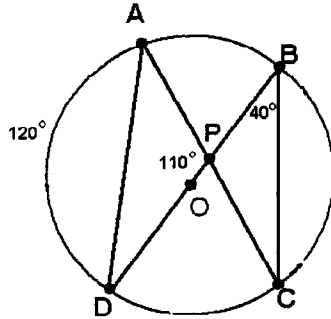
- (A) 3 (B) 2 (C) 0 (D) -2 (E) -3

40. The graph of $x^2 - 2xy + y^2 - 7x + 7y + 10 = 0$ is a(n) _____.

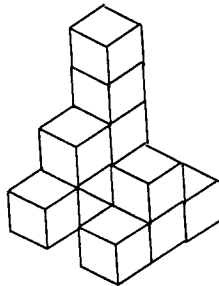
- (A) point (B) line (C) pair of parallel lines (D) ellipse (E) hyperbola

41. Robin D. Hood shoots an arrow from an 80 ft castle wall at a velocity of 64 ft/sec. Its height, H in feet, above the ground at S seconds can be found by the equation $H = 80 + 64S - 16S^2$. Find the instantaneous velocity at 1.5 seconds.
- (A) 140 ft/sec (B) 112 ft/sec (C) 48 ft/sec (D) 20 ft/sec (E) 16 ft/sec
42. The following cards are removed from a standard deck of 52 cards: $2\spadesuit, 4\spadesuit, 6\spadesuit, 8\spadesuit, 10\spadesuit, 3\clubsuit, 5\clubsuit, 7\clubsuit,$ and $9\clubsuit$. The remaining cards are shuffled. Two cards are dealt face up. What is the probability that the first card is a $J\spadesuit, Q\spadesuit,$ or $K\spadesuit$ and the second card is a club? (nearest tenth)
- (A) 5.0% (B) 3.5% (C) 2.8% (D) 2.3% (E) 1.5%
43. Lotta Moolaw went to the bank and got some \$1 bills, \$5 bills, \$10 bills, \$20 bills, \$50 bills, and \$100 bills. She is stuffing donation envelopes with 4 bills. How many different envelopes containing 4 bills can Lotta make?
- (A) 21 (B) 840 (C) 126 (D) 84 (E) 210
44. Which of the following numbers is considered to be a *lucky prime* number that is *odious*?
- (A) 2 (B) 3 (C) 9 (D) 11 (E) 13
45. Willie Proffett bought 3 used computers at a garage sale for \$200 each. He sold his neighbor one for a 30% profit and sold another one to his favorite uncle for half of his cost. How much will he have to sell the third one for to make a net profit of 25%?
- (A) \$240.00 (B) \$255.00 (C) \$305.00 (D) \$345.00 (E) \$390.00
46. If Lois Skor scores 246 on her next UIL math test, her average score will be 264. If she scores 276, her average score will be 267. How many tests had she already taken?
- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
47. The Real value solution set for $|5 - 3x| > 2$ is?
- (A) $\{x \mid \{2\frac{2}{3} > x\} \cup \{1 > x\}\}$ (B) $\{x \mid 1 < x < 2\frac{2}{3}\}$ (C) $\{x \mid \{x > 1\} \cup \{x < 2\frac{1}{3}\}\}$
 (D) $\{x \mid \{x \mid 1 > x > 2\frac{2}{3}\}\}$ (E) $\{x \mid \{x < 1\} \cup \{x > 2\frac{1}{3}\}\}$
48. The solution set of $6x - 5y = 30$ is $\{(x, y) \mid x, y \in \{\text{Integers}\}, x \geq 0, \text{ and } y \leq 0\}$. How many such ordered pairs exist?
- (A) 6 (B) 5 (C) 3 (D) 2 (E) 0

49. \overline{AC} , \overline{AD} , \overline{BD} , and \overline{BC} are chords of circle O and intersect at point P. Find $m\widehat{AB}$.

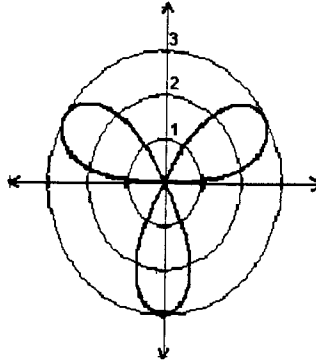


- (A) 35° (B) 50° (C) 60° (D) 70° (E) 80°
50. Find the area of a circumscribed circle of a triangle of sides 5", 12", and 13". (nearest tenth)
- (A) 265.5 sq. in (B) 113.1 sq. in (C) 153.9 sq. in (D) 115.5 sq. in (E) 132.7 sq. in
51. Simplify: $[\log_a(6k) - \log_a(4)] + [\log_a(3) - \log_a(5k)]$
- (A) $\log_a(9) - \log_a(10)$ (B) $2\log_a(22.5k)$ (C) $\log_a(1.6)$
- (D) $\log_a(k) - \log_a(1)$ (E) $\log_a(11k)$
52. Thirteen congruent cubes are glued together to form the figure shown. Dee Artiste picks up the figure and paints each of the cubes' faces blue that are not glued together. How many of the cubes' faces will she paint blue?



- (A) 49 (B) 48 (C) 47 (D) 46 (E) 45
53. A sine function, $f(x)$, has a maximum value of 7, a minimum value of 3, a period of $\frac{2\pi}{3}$ and a horizontal phase shift of $\frac{\pi}{2}$. Determine the value of $f(\frac{\pi}{3})$. (nearest tenth)
- (A) 5.2 (B) 5.0 (C) 4.5 (D) 3.4 (E) 3.0
54. If the three numbers 433, 342, and 290 are each divided by the number D, each of their quotients will have the same remainder R. Find R.
- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

55. Which of the following polar equations will produce this graph on a polar grid?



- (A) $r = 3\cos(\theta) + 3$ (B) $r = \sin(3\theta) + 3$ (C) $r = 3\cos(3\theta)$
 (D) $r = 3\sin(3\theta)$ (E) $r = 3\sin(\theta) - 3$

56. Find C if the remainder when $x^3 - 9x^2 + 15x + C$ is divided by $x - 3$ is -14 .

- (A) -12 (B) -9 (C) -5 (D) 3 (E) 15

57. Evaluate: $\prod_{n=2}^5 \left(\frac{1}{n} - n\right)$

- (A) $129\frac{3}{5}$ (B) 72 (C) 15 (D) $-12\frac{43}{60}$ (E) -15

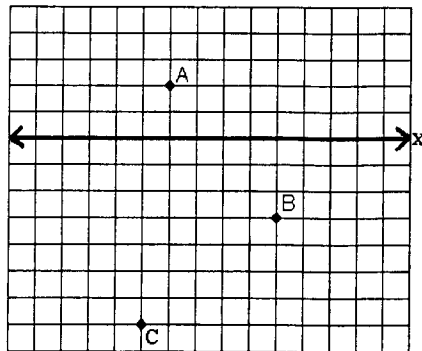
58. Larry, Moe, Curly, and four of the little rascals are randomly assigned seats in a row of seven chairs. What is the probability that Larry, Moe, and Curly will be seated in no specific order next to each other? (nearest tenth)

- (A) 42.9% (B) 25.0% (C) 14.3% (D) 7.1% (E) 2.4%

59. The U-Nee-Cue tribe uses a special base to solve their math problems. Using their special base they found that $16Q - 70 = 0$ and $13Q - 60 = 0$. What base are they using?

- (A) base 5 (B) base 8 (C) base 12 (D) base 15 (E) base 16

60. The x-axis of a Cartesian Coordinate plane is shown. Point A is in QII, the coordinates of point B is $(1, -3)$, and the coordinates of point C is (x, y) . Find $x - y$.



- (A) 10 (B) 4 (C) 3 (D) -5 (E) -11

**University Interscholastic League
MATHEMATICS CONTEST
HS • State • 2010
Answer Key**

- | | | |
|-------|-------|-------|
| 1. C | 21. D | 41. E |
| 2. B | 22. A | 42. E |
| 3. A | 23. D | 43. C |
| 4. C | 24. E | 44. E |
| 5. C | 25. A | 45. E |
| 6. B | 26. D | 46. D |
| 7. D | 27. C | 47. E |
| 8. A | 28. B | 48. D |
| 9. B | 29. A | 49. C |
| 10. C | 30. E | 50. E |
| 11. E | 31. D | 51. A |
| 12. C | 32. E | 52. B |
| 13. C | 33. D | 53. E |
| 14. E | 34. D | 54. C |
| 15. B | 35. C | 55. D |
| 16. D | 36. D | 56. C |
| 17. E | 37. A | 57. B |
| 18. A | 38. E | 58. C |
| 19. C | 39. C | 59. D |
| 20. A | 40. C | 60. C |