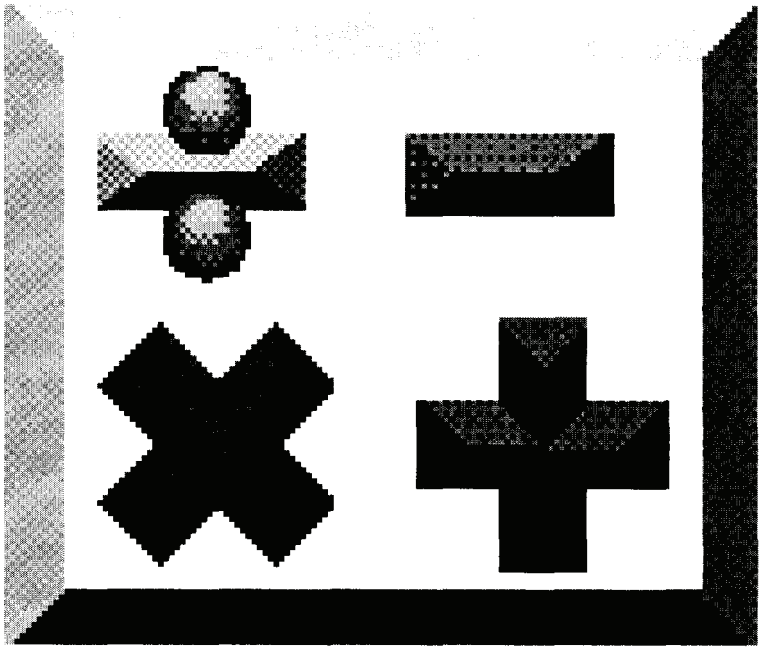
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Regional • 2010



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

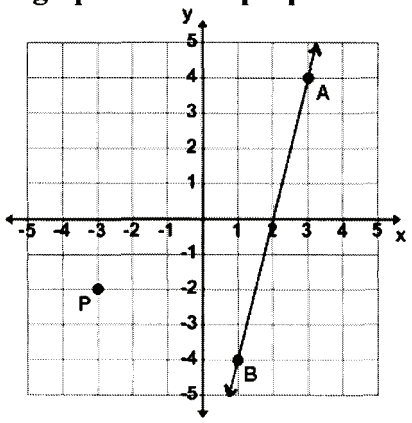
1. Evaluate: $\left[4! - (3)^3\right] + 2^{-2} \times \sqrt{2^4 \div 3^4}$

(A) $-3\frac{1}{9}$ (B) $-2\frac{8}{9}$ (C) $-1\frac{2}{9}$ (D) $6\frac{7}{9}$ (E) $15\frac{1}{9}$

2. Will Itkosmoor wants to buy 4 new calculators for his math team. He can buy 2 at the regular price, 2 at half price, and pay 8% of the total price for shipping and handling. He can get 16% off and pay no shipping if he buys 4 at the regular price. If the regular price is $89.95, how much will he save if he takes the best deal? (tax exempt)

(A) $10.79 (B) $10.30 (C) $9.59 (D) $7.20 (E) $5.40

3. Find an equation of a line through point P and perpendicular to the line shown.



(A) $x + 4y = -14$ (B) $4x - y = 14$ (C) $x - 4y = 5$ (D) $x + 4y = -11$ (E) $4x + y = 5$

4. The statement "If $x = y + 1$ then $y + 1 = x$" is an example of the ____ property of real numbers?

(A) commutative (B) inverse (C) reflexive (D) identity (E) symmetric

5. The length of a rectangle is increased 40%. The width of the rectangle is decreased by 60%. The area of the new rectangle is what fractional part of the area of the original rectangle?

(A) $\frac{21}{25}$ (B) $\frac{14}{25}$ (C) $\frac{11}{25}$ (D) $\frac{9}{25}$ (E) $\frac{7}{25}$

6. Find the area, in square units, of the quadrilateral whose vertices are $(-3, -3)$, $(3, -2)$, $(0, 2)$, and $(-2, 1)$.

(A) 14 (B) 15.5 (C) 16 (D) 17 (E) 19.5

7. Noah Sense has 42 coins consisting of pennies, nickels, dimes, and quarters. He has twice as many nickels as pennies, three less dimes than nickels, and three more quarters than pennies. How much money does he have?

(A) $ 5.31 (B) $ 4.37 (C) $ 4.20 (D) $ 4.07 (E) $ 3.81

8. The graph of $x^2 + y^2 - 10x + 6y + 9 = 0$ is a circle with a center (h, k) and a radius r. Find $h + k + r$.

(A) 1 (B) 3 (C) 7 (D) 13 (E) 15

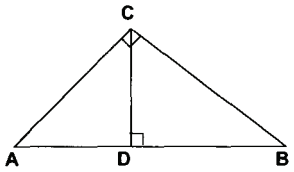
9. Using the equation $y = 4 + 3\sin(2x + 1)$, which of the following has the smallest numeric value?

(A) amplitude (B) displacement (C) frequency (D) period (E) phase shift

10. Which of the following is equivalent to $\frac{\sin(\theta)}{1 + \cos(\theta)} + \frac{1 + \cos(\theta)}{\sin(\theta)}$?

(A) $\csc(\theta) + 1$ (B) $\frac{\sec(\theta)}{2}$ (C) $\cot(\theta)$ (D) $\tan(\theta)$ (E) $2\csc(\theta)$

11. Find BC if AD = 24 cm. and DB = 30 cm. (nearest tenth)



(A) 49.4 cm (B) 36.0 cm (C) 44.9 cm (D) 26.8 cm (E) 40.2 cm

12. Let $f(x) = 3x - 2$, and $g(x) = 2x + 1$. Find the $f(f(-x)) - g(g(-x))$.

(A) $-7 - 13x$ (B) $-11 - 5x$ (C) $7 - 5x$ (D) $4 - 9x$ (E) $3 - 4x$

13. Find the angle of rotation, θ (nearest tenth degree), where $0° < \theta < 90°$, such that the conic $2x^2 + 12xy + 18y^2 - 3y = 5$ contains no xy term in its equation.

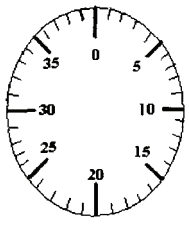
(A) 74.5° (B) 71.6° (C) 63.4° (D) 60.5° (E) 58.6°

14. $\int \left(\frac{3-x}{x+2}\right) dx =$ _____ + C, where C is some arbitrary constant.

(A) $x - 3\ln(x + 2)$ (B) $5\ln(x - 2) + x$ (C) $x + 3\ln(x - 2)$

(D) $5\ln(x + 2) - x$ (E) $5\ln(x + 2) + x$

15. E. Z. Lockett forgot her 3 number combination to the padlock shown. She knows that all of the numbers have a 3 as one of its digits and all 3 numbers of the combination are different. How many combinations can she try to open the lock?

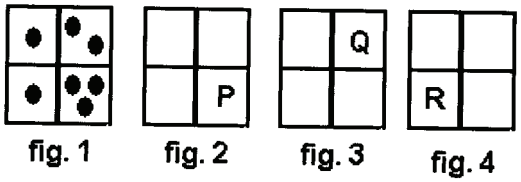


(A) 1,320 (B) 2,184 (C) 1,716 (D) 1,872 (E) 2,197

16. The *Play Ball* Association is putting 5 balls in each gift box for kids to play with. The association has golf balls, baseballs, basketballs, footballs, ping pong balls, tennis balls, and dodge balls. How many different gift boxes of 5 balls can they put together?

(A) 13,860 (B) 5,544 (C) 792 (D) 462 (E) 330

17. Figure 1 is a square with four smaller squares, each containing a number of dots. Reflect fig. 1 over its vertical axis. The reflected figure will be fig. 2. Then, translate fig. 2 horizontally. The translated figure will be fig. 3. Finally, rotate fig. 3 180 ° clockwise. The rotated figure will be fig. 4. What is the total number of dots in the smaller squares denoted by P, Q, & R?



(A) 7 (B) 6 (C) 5 (D) 4 (E) 3

18. Which of the following mathematicians is known for their work explaining and clarifying the the number $\frac{1+\sqrt{5}}{2}$?

(A) Theano of Crotona (B) Grace Alele Williams (C) Hypatia

(D) Agnesi (E) Freda Porter

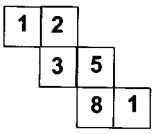
19. Ester Bunnee had a bag of chicken peeps. She hid 4 of them under a bush. She gave 25% of what was left to her cousin Dee Hair. The she gave $\frac{5}{6}$ of what was left to the little boys and girls. She had 2 left for herself. How many peeps were in the bag to begin with?

(A) 32 (B) 24 (C) 40 (D) 36 (E) 20

20. The number 4321 in base 5 is equivalent to the number wxyz in base 7, where w, x, y, and z are digits. Find $w + x + y + z$.

(A) 18 (B) 16 (C) 11 (D) 10 (E) 9

21. When the net below is folded into a cube the sum of the faces opposite the faces containing the number 1 will be?



(A) 5 (B) 7 (C) 8 (D) 10 (E) 11

22. Let p and q be the real roots of $x^2 - 2x - 8 = 0$, where $p > q$. Find $p^3q + 2p^2q^2 + pq^3$.

(A) -32 (B) -16 (C) 4 (D) 6 (E) 8

23. Bea Debest, Ima Slo, and Betsy Luzes run in a 200 meter race. When Bea crosses the finish line, Ima is 10 meters behind Bea. When Ima crosses the finish line, Betsy is 10 meters behind Ima. If all 3 runners ran at a constant speed, how far was Betsy from the finish line when Bea won the race?

(A) 18 meters (B) 19 meters (C) 19.5 meters (D) 20 meters (E) 21.5 meters

24. If y varies directly as x and inversely as z, then $z = 4$ when $x = 2$ and $y = 5$. Find z when $x = 5$ and $y = 3$.

(A) $33\frac{1}{3}$ (B) $16\frac{2}{3}$ (C) 6 (D) $4\frac{1}{6}$ (E) $\frac{1}{6}$

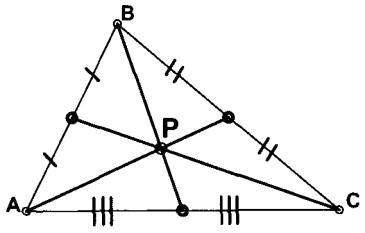
25. Points A, B, and D are on circle O. $\overline{CA}$ is secant to O through point B. $\overrightarrow{CD}$ is tangent to O at D. If $m\overset{\frown}{AD} = 80°$ and $m\overset{\frown}{BD} = 30°$, then $m\angle BCD = ?$

(A) 15 ° (B) 25 ° (C) 40 ° (D) 55 ° (E) 110 °

26. A box contains five rods whose lengths are 4", 5", 7", 11" and 12". How many different acute triangles can be made using only three rods at a time.

(A) 7 (B) 5 (C) 4 (D) 2 (E) 0

27. Point P is the _____ of $\triangle ABC$ shown below.



(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) center

28. Let $f(x) = x^3 + 3$ and $g(x) = x^2 - 2$ and $h(x) = x + 1$. Find $f\big(g\big(h(-2)\big)\big)$.

(A) 2 (B) 4 (C) 12 (D) 24 (E) 30

29. If you slice a complete cone (double cone) with a plane through the diameter of the cone and its vertex point, the intersection is a _____ .

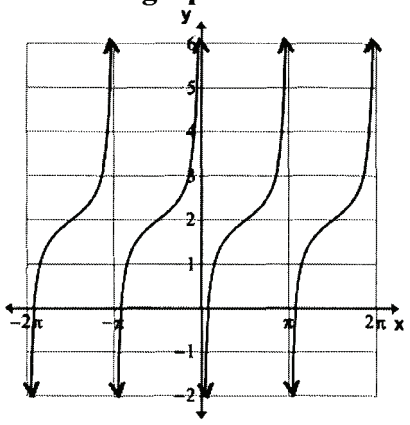
(A) line (B) pair of intersecting lines (C) pair of parallel lines

(D) pair of perpendicular lines (E) point

30. If $a_1 = 2$, $a_2 = -1$, $a_3 = 1$ and $a_n = (a_{n-3})(a_{n-2}) - a_{n-1}$, where $n \geq 4$, then a_7 equals:

(A) 9 (B) 5 (C) 2 (D) -1 (E) -11

31. The equation $y =$ _____ will produce this graph.



(A) $1 + 2\tan(x)$ (B) $\frac{4 - \cot(x)}{2}$ (C) $\frac{1 - 2\tan(x)}{2}$ (D) $2 - \cot(x)$ (E) $\frac{2 + 4\cot(x)}{2}$

32. The type of graph of the polar equation $r^2 = 25\sin(2\theta)$ is called a:

(A) Archimedian spiral (B) cardiod (C) lemniscate (D) limacon (E) rose

33. How many elements are in $\left\{x \mid 2 + \csc(2x - \pi) = 0, x \in \left([-\pi, 0] \cup [\pi, 2\pi]\right)\right\}$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

34. Let $f(x) = \cos(x)\sin(x)$ for all Real numbers. Which of the following is true about f(x)?

(A) It is an odd function. (B) It is an even function. (C) It has two asymptotes.

(D) It is neither an even nor an odd function (E) It is a one-to-one function.

35. Let $A = \begin{bmatrix} 1 & -2 \\ 0 & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 4 \\ y & 2 \end{bmatrix}$. Find $x + y$.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) 1 (E) -4

36. How many asymptotes exist of $h(x) = \frac{x + 10}{|x|}$?

(A) none exists (B) 1 (C) 2 (D) 3 (E) 4

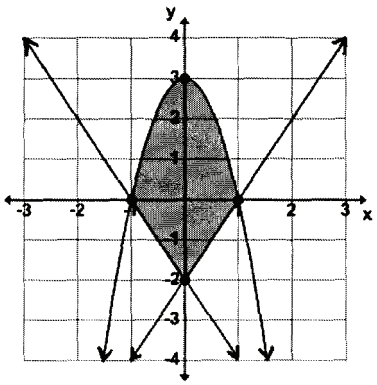
37. Let $f(x) = 3x^2 - 4x + 3$. A line tangent to f(x) at $x = 0$ intersects with a line tangent to f(x) at $x = 2$ at point (x, y). Find y.

(A) 0.8 (B) 0.5 (C) 0 (D) -0.2 (E) -1

38. If $f''(x) = 12x + 2$ and $f'(1) = 5$ and $f(1) = 4$, then $f(-1) =$ _____.

(A) 6 (B) 3 (C) 1 (D) -4 (E) -10

39. **Find the area of the shaded region in square units.**



(A) 6 (B) $5\frac{3}{4}$ (C) $6\frac{1}{2}$ (D) $5\frac{1}{4}$ (E) 5

40. **The Brite Lite Company produced 5000 l00-watt bulbs of which 50 were defective. The Brite Bulb Company produced 3000 100-watt bulbs of which 100 were defective. A bulb was chosen at random from the 8000 bulbs and turns out to be defective. What is the probability that the bulb came from the Brite Lite Company?**

(A) $33\frac{1}{3}\%$ (B) $18\frac{3}{4}\%$ (C) 10% (D) $3\frac{1}{3}\%$ (E) 1%

41. **A pair of dice are rolled. What are the odds that the roll comes up a 2, 5, 6, 10, or 12?**

(A) 7 to 18 (B) 5 to 7 (C) 5 to 12 (D) 7 to 12 (E) 7 to 11

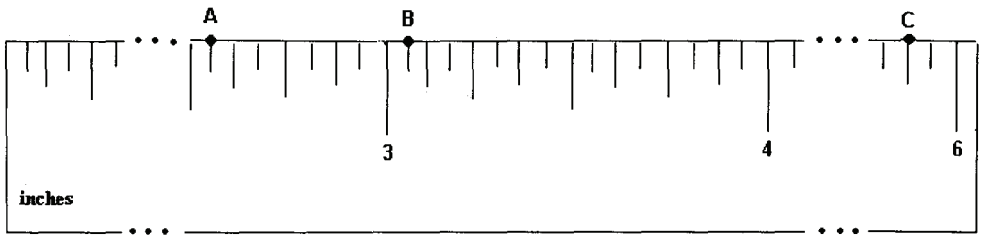
42. **Let** $x = \cfrac{2}{3+\cfrac{2}{4+\cfrac{2}{3+\cfrac{2}{4+\ldots}}}}$ **be the continued fraction. Find x. (nearest tenth)**

(A) 4.6 (B) 1.7 (C) 1.2 (D) 0.6 (E) 0.3

43. **The operation m ★ n is defined as $(m + n) \div (m \times n)$. Compute (1 ★ 9) ★ (3 ★ 3).**

(A) 6 (B) 0.444... (C) 2.111 (D) 2.4 (E) 1

44. **May Juror uses a 6" ruler to find the lengths of three pieces of string. One piece has a length of A, a second piece has length B, and a third piece has a length of C. What is the average length of the three pieces of string?**



(A) $3\frac{1}{3}$ " (B) $3\frac{3}{8}$ " (C) $3\frac{5}{6}$ " (D) $3\frac{15}{16}$ " (E) $3\frac{27}{32}$ "

45. Let F = { 1, 2, 3, 5, 8, 13, 21}, P = { 1, 3, 4, 7, 11, 18 }, and H = { 2, 3, 4, 5, 6, 8, 9}. How many elements are in $P \cup (F \cap H)$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

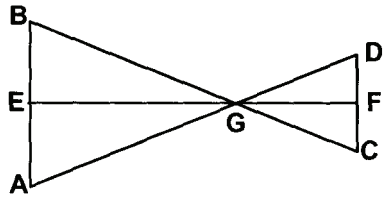
46. Anne Surr's final exam is worth 120 points. The exam consists of 45 problems of which some problems are worth 2 points and the others are worth 3 points. Find Anne's score on the test if she got all of the 3-pointers correct and missed all of the 2-pointers.

(A) 108 (B) 105 (C) 99 (D) 90 (E) 84

47. Simplify: $\frac{9x^2-1}{3x^2+4x+1} \div \frac{3x^2-10x+3}{9x^2+6x+1}$

(A) $\frac{3x+1}{x+1}$ (B) $\frac{x^2-2x-3}{3x+1}$ (C) $\frac{3x+1}{x-3}$ (D) $\frac{x+1}{(3x+1)^2}$ (E) $\frac{(3x+1)^2}{x^2-2x-3}$

48. $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \perp \overline{EG}$, and $\overline{CD} \perp \overline{FG}$. If AB = 27, EG = 21, and FG = 14, then CD = ?



(A) 40.5 (B) 20 (C) 18 (D) 10.888... (E) not enough information given

49. Let $\triangle PQR$ be a right triangle with $\overline{QR}$ being the hypotenuse and point M the midpoint of $\overline{QR}$. Which of the following is a true statement?

(A) $MQ + MR = (PQ + PR)^2$ (B) $MP = PQ \div 2$ (C) $MP = MQ = MR$

(D) $MQ + MR = (PQ)(PR) \div 2$ (E) $MP = (PQ + PR) \div 2$

50. If $\sqrt[4]{x^3\left(\sqrt[5]{x^4\left(\sqrt[3]{x^2}\right)}\right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then n + k = ?

(A) 21 (B) 38 (C) 69 (D) 84 (E) 119

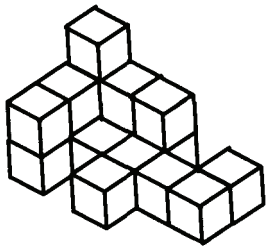
51. Juan Weeler rides his unicycle at 10 mph in the local parade. The radius of the wheel is 18 inches. What is the angular velocity of the unicycle wheel in radians per minute? (nearest tenth)

(A) 586.7 (B) 560.2 (C) 186.8 (D) 176.0 (E) 93.4

52. If the three numbers 259, 223, and 196 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

(A) 2 (B) 3 (C) 4 (D) 7 (E) 9

53. One-centimeter cubes are glued together to form the object in the figure shown. Art Payntor picks up the figure and paints each of the cubes' faces red that are not glued together. How many of the cubes' faces will he paint red?



(A) 56 (B) 57 (C) 58 (D) 59 (E) 60

54. Find the direction of a resultant vector whose vertical component has a magnitude of 9 and a direction of 90 ° and a horizontal component having a magnitude of 6 and a direction of 180 °. (nearest tenth)

(A) 156.3 ° (B) 146.3 ° (C) 133.7 ° (D) 123.7 ° (E) 112.6 °

55. The polar graph of $r = 2\sin(3\theta)$ is symmetric to the:

(A) polar axis (B) pole (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{4}$ (E) line $\theta = \pi$

56. The eccentricity of the ellipse $\frac{(x-3)^2}{49} + \frac{(y+1)^2}{25} = 1$ is:

(A) $\frac{\sqrt{24}}{5}$ (B) $\frac{24}{25}$ (C) $\frac{\sqrt{24}}{25}$ (D) $\frac{5}{7}$ (E) $\frac{\sqrt{24}}{7}$

57. Evaluate: $\prod_{n=2}^{6} (n - \frac{1}{n})$

(A) 420 (B) 74.2 (C) 72 (D) 36 (E) 18.55

58. The president wants to form a finance committee consisting of 3 Democrats, 2 Republicans, 1 Libertarian, and 1 Independent. He can choose this committee from a group of 9 Democrats, 7 Republicans, 5 Libertarians, and 3 Independents. How many different committees can he form?

(A) 346,104 (B) 26,460 (C) 168 (D) 29,400 (E) 113

59. Simplify: $\frac{n!\,(n-1)!\,(n+2)!}{(n+1)!\,(n-2)!}$

(A) $(n+1)!$ (B) $n!(n^2+n-2)$ (C) $n!\,(n-1)$ (D) $n!$ (E) $n!\,(n+2)$

60. How many of the elements in the set {6, 28, 496, 8128, 12468} are considered to be both odious and perfect numbers?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2010
Answer Key

1. B	21. C	41. E
2. A	22. A	42. D
3. D	23. C	43. D
4. E	24. B	44. C
5. B	25. B	45. D
6. D	26. D	46. D
7. B	27. A	47. E
8. C	28. A	48. C
9. E	29. B	49. C
10. E	30. D	50. E
11. E	31. B	51. A
12. B	32. C	52. D
13. B	33. C	53. C
14. D	34. A	54. D
15. C	35. A	55. C
16. D	36. D	56. E
17. E	37. E	57. A
18. A	38. A	58. B
19. E	39. A	59. B
20. B	40. A	60. C