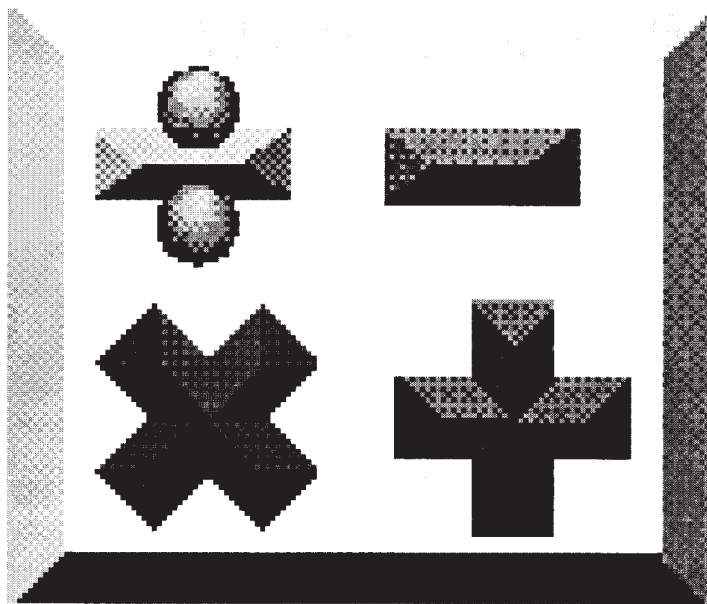UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics
District 2 • 2010

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $6 \times 1\frac{1}{3} + 8 \div 1\frac{3}{5} - 10 \div 1\frac{5}{7}$

   (A) $4\frac{1}{6}$ (B) $6\frac{4}{5}$ (C) $7\frac{1}{6}$ (D) $10\frac{3}{5}$ (E) $18\frac{5}{6}$

2. The original price of a suit at the *Everything-Must-Go* thrift store was \$89.99. It was successively discounted by 20%, then 10%, and finally 5%. What was the price of the suit after the three discounts? (nearest cent)

   (A) \$45.00   (B) \$55.01   (C) \$58.49   (D) \$61.55   (E) \$74.62

3. Find an equation of the line through point P and perpendicular to the line shown.
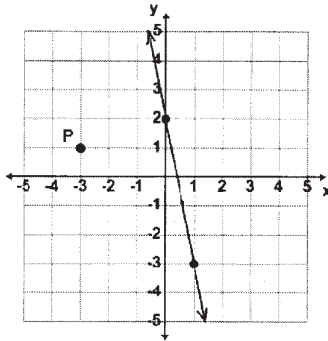
   (A) $x + 5y = 2$ (B) $x - 5y = -8$ (C) $2x - 5y = -11$ (D) $2x + 5y = -1$ (E) $5x - 2y = 1$

4. The distance from El Paso to San Antonio by way of I-10 is 545 miles. Speedy Karr is leaving El Paso on I-10 at 7:00 a.m. driving toward San Antonio at 80 mph. Wes Bound is leaving San Antonio on I-10 at the same time driving toward El Paso at 65 mph. How much farther will Speedy have driven than Wes when they meet? (nearest mile)

   (A) 56 miles   (B) 73 miles   (C) 15 miles   (D) 45 miles   (E) 36 miles

5. The length of a rectangle is decreased by 25%. The width of the rectangle is increased by 25%. The area of the original rectangle is what percent less than the area of new rectangle?

   (A) $12\frac{1}{2}\%$ (B) $9\frac{3}{8}\%$ (C) $8\frac{3}{4}\%$ (D) 6% (E) $6\frac{1}{4}\%$

6. A segment whose length is the shortest distance from the center to the side of a regular polygon is called a(n):

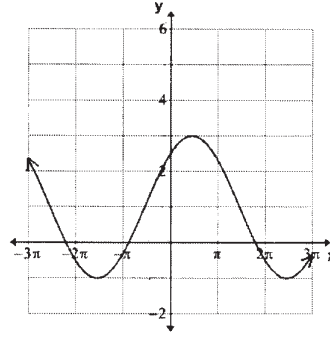   (A) tangent   (B) apothem   (C) arc length   (D) secant   (E) altitude

7. The roots of the equation $6x^3 + kx^2 - 5x + 6 = 0$ are $0.5, 3,$ and R. Find k.

   (A) -3 (B) -12 (C) 7.5 (D) -17 (E) 15

8. If $\dfrac{11}{(x-2)(x-3)} - \dfrac{7}{(x-3)(x-5)} = \dfrac{k}{(x-2)(x-3)(x-5)}$, then k equals:

   (A) $4x + 13$ (B) $4x - 9$ (C) $4x + 1$ (D) $4x - 41$ (E) $4x - 19$

9. The equation $y =$ _____ will produce this graph.

(A) $1 + 2\cos(\frac{x}{2} - 4)$ (B) $1 - 2\sin(\frac{x}{4} + 2)$ (C) $1 + 2\cos(\frac{x}{4} - 2)$

(D) $2\sin(\frac{x}{2} - 4) + 1$ (E) $1 - 2\sin(\frac{x}{2} + 4)$

10. Determine the frequency of $y = 4 + 3\tan(2\pi x + 1)$.

(A) 4   (B) 3   (C) 2   (D) $1\frac{1}{2}$ (E) $\frac{1}{2}$

11. The function $f(x) = 4x^3 - 12x^2 - 5x + 1$ has at most ____ negative real zeros.
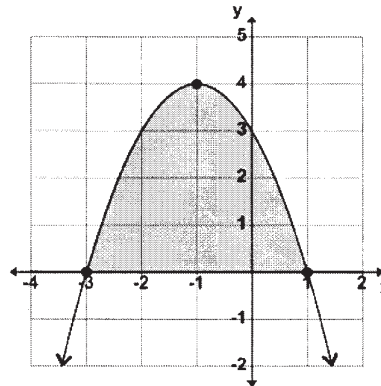
(A) 0   (B) 1   (C) 2   (D) 3   (E) 4

12. Find the determinant of the 3 x 3 matrix.

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 3 \\ -3 & 2 & 1 \end{bmatrix}$$

(A) 0   (B) -6 (C) -14 (D) -35 (E) -42

13. Find the area of the shaded region in square units.

(A) 11   (B) $10\frac{2}{3}$ (C) $10\frac{1}{2}$ (D) $10\frac{1}{3}$ (E) 10

14. If $f''(x) = 10$ and $f'(1) = 7$ and $f(-1) = 10$, then $f(2) =$ ____.

(A) 28   (B) 21   (C) 17   (D) 16   (E) 13

15. How many ways can a father, mother, son, and daughter be seated around a circular table with 4 chairs?

(A) 6   (B) 10   (C) 12   (D) 18   (E) 24

16. All of the aces, treys, fives, sevens, and nines are removed from a standard 52-card deck. The remaining cards are shuffled. Two cards are dealt face up. What is the probability that the first card is a face card and the second card is a factor of 8? (nearest %)

(A) 38%   (B) 25%   (C) 20%   (D) 15%   (E) 13%

17. $1100_2 - 123_4 + 67_8 = $ _____ $_{16}$

(A) 15   (B) 28   (C) 32   (D) 40   (E) 65

18. Fig. 1 is rotated clockwise 270 ° to fig. 2. Then, fig. 2 is reflected over its positive diagonal to fig. 3. Then fig. 3 is translated to the right to fig. 4. What is the total number of dots in squares P, Q, & R?

fig. 1   fig. 2   fig. 3   fig. 4
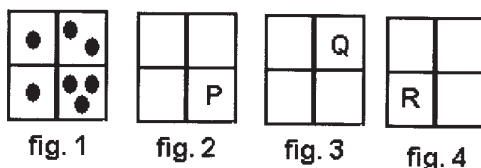
(A) 7   (B) 6   (C) 5   (D) 4   (E) 3

19. The number 11111 in base 2 is equivalent to the number X in base 5. Find the sum of the digits in the number X.
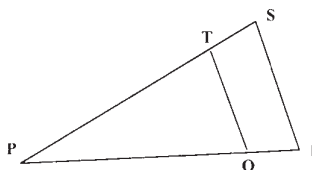
(A) 2   (B) 3   (C) 4   (D) 5   (E) 6

20. On the map legend, $\frac{1}{4}$ inch represents 11 miles. The town of Fife is 143 miles from San Angelo. How far is it on the map?

(A) $2\frac{3}{4}$ "   (B) $2\frac{7}{8}$ "   (C) 3 "   (D) $3\frac{1}{4}$ "   (E) $3\frac{5}{8}$ "

21. Which of the following is an example of the inverse property of real numbers?

(A) $a - 0 = a$ (B) $b \times \frac{1}{b} = 1$ (C) $c + 0 = c \times 1$ (D) $0 \div d = 0$ (E) $e = e$

22. $\triangle$PRS is similar to $\triangle$PQT. PT = 6, ST = 3, RS = x + 1, and QT = x — 2. Find x.

(A) 2   (B) 4   (C) 8   (D) 9   (E) not enough information given

23. Eve N. Steevan bought 2 television sets. Each set cost the same price. She sold one of them for 80% of her purchase price. What percentage of the cost of the other set must she sell it for in order to make a 20% profit of the total original cost of both sets?

(A) 110%   (B) 120%   (C) 140%   (D) 150%   (E) 160%

24. The adjacent dots on the grid are 5 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown. (nearest tenth)
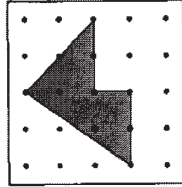

(A) 91.0 cm^2 (B) 112.5 cm^2 (C) 122.5 cm^2 (D) 125.0 cm^2 (E) 137.5 cm^2

25. Find $f(-1) + f(0) - f(2)$ if $f(x) = \begin{cases} x - 1 & \text{if } x < 0 \\ x & \text{if } x = 0 \\ 1 + x & \text{if } x > 0 \end{cases}$

(A) -5 (B) -2 (C) 0   (D) 1   (E) 3

26. Let p and q be the roots of $x^2 - 9x + 20 = 0$, where $p > q$.
Find $p^5 - 5p^4q + 10p^3q^2 - 10p^2q^3 + 5pq^4 - q^5$.

(A) -5 (B) -1 (C) 1   (D) 5   (E) 9

27. A water tank in the shape of a right cylinder is 6 feet high and has an inside diameter of 8 feet. The tank is not full. It contains 1000 gallons of water. How deep is the water? (nearest inch)

(A) 2' 3 "   (B) 2' 6 "   (C) 2' 8 "   (D) 3' 1 "   (E) 3' 7 "

28. If $a_1 = 2$, $a_2 = -3$, $a_3 = 5$ and $a_n = (a_{n-2})(a_{n-3}) + a_{n-1}$, where $n \geq 4$, then a_7 equals:

(A) -21 (B) -16 (C) -11 (D) -5 (E) -1

29. Let $f(x) = 1 + 2x$ and $g(x) = 3 - 4x$. If $h(x)$ is the inverse function of $\frac{f(x)}{g(x)}$, then $h(5) = ?$

(A) $-1\frac{6}{11}$ (B) $-1\frac{1}{9}$ (C) $-\frac{11}{17}$ (D) $\frac{8}{9}$ (E) $\frac{7}{11}$

30. If $\sqrt{x^3 \left(\sqrt[3]{x^2 \left(\sqrt[4]{x} \right)} \right)} = \sqrt[n]{x^k}$, where k and n are relatively prime, then $k = ?$

(A) 18   (B) 15   (C) 8   (D) 5   (E) 4

31. How many elements are in $\left\{x \mid 3\sin(x)\cos(\frac{\pi}{3}) + 3\cos(x)\sin(\frac{\pi}{3}) = 0, x \in (-\pi, 2\pi)\right\}$?

   (A) 4   (B) 3   (C) 2   (D) 1   (E) 0

32. Clark and Lois are camped on opposite sides of a small lake. Straight paths from each of their camps lead to the Meet & Greet Bait store. Clark walks 600 feet to the store and Lois walks 870 feet to the store. The angle between the two paths is 35 °. What is the distance between the two camps straight across the lake?   (nearest foot)

   (A) 512 ft   (B) 540 ft   (C) 605 ft   (D) 718 ft   (E) 735 ft

33. Find the sum of the series $5 - \frac{(5)^3}{3!} + \frac{(5)^5}{5!} - \frac{(5)^7}{7!} + \frac{(5)^9}{9!} - \ldots$ (nearest ten-thousandth)

   (A) -1.1336 (B) -0.9376 (C) -0.9589 (D) 0.0896   (E) 0.9753

34. The directrix of the conic given by the equation $y = -2x^2 + 5x - 3$ is:

   (A) $y = \frac{1}{8}$ (B) $y = \frac{1}{4}$ (C) $y = \frac{5}{16}$ (D) $y = \frac{11}{16}$ (E) $y = \frac{5}{4}$

35. Let $p + q = 12$, where $p, q > 0$. Find p and q such that pq^2 is a maximum product.

   (A) 128   (B) 200   (C) 243   (D) 245   (E) 256

36. Eight 400 meter relay teams are competing at the Eden Spring Track Meet. In how many ways can the first, second, and third place trophies be awarded?

   (A) 42   (B) 56   (C) 171   (D) 336   (E) 512

37. Ma Bell's telephone company assigns 10 digit phone numbers (i.e. 934-227-1458). What are the odds that the last digit of a randomly chosen phone number is 9 or 0?

   (A) 1 to 5   (B) 1 to 4   (C) 1 to 10   (D) 5 to 1   (E) 4 to 1

38. The odd numbers from 1 to 17 are to be placed in this magic square in which the rows, the columns, and the diagonals have the same sum. Find the value of x.

	13	
17		1
		x

   (A) 7   (B) 9   (C) 11   (D) 15   (E) 19

39. Lotta Sense has 6 coins, a penny, a nickel, a dime, a quarter, a half-dollar, and a silver dollar. How many different sets of three coins can she create?

   (A) 20   (B) 18   (C) 15   (D) 12   (E) 9

40. Let $\triangle ABC$ be an isosceles right triangle and $m\angle BAC = 90\,°$. If the orthocenter of the triangle is the vertex point A, which of the following lies on segment BC?

   (A) centroid   (B) circumcenter   (C) incenter   (D) outcenter   (E) midcenter

41. The mathematician who came up with a type of linguistic algebra using the three most basic operations AND, OR, and NOT and developed a logic system based on a binary approach (yes-no, on-off, true-false, 0-1) is:

   (A) John Venn   (B) George Boole   (C) John Napier
   (D) Leonard Euler   (E) Claudius Ptolemy

42. The value of $58\frac{1}{3}\%$ of $(0.777\ ...)^{-1}$ is the same as the value of 25% of _____.

   (A) 3   (B) $1\frac{22}{27}$ (C) 1.5   (D) .1875   (E) $\frac{1}{48}$

43. Let $L = \{\,2, 1, 3, 4, 7, 11\}$, $P = \{\,2, 3, 5, 7, 11\,\}$, and $O = \{\,1, 3, 5, 7, 9, 11\}$. How many elements are in $(L \cap P) \cup (L \cap O) \cup (P \cap O)$?

   (A) 2   (B) 4   (C) 6   (D) 8   (E) 10

44. $\left\{(x\,,\,y)\mid x,y \in \{\text{Integers}\}, -10 \le x \le 10, \text{ and} -5 \le y \le 5\right\}$ is the solution set of $3x - 5y = 7$. How many such ordered pairs exist?

   (A) 6   (B) 5   (C) 4   (D) 3   (E) 2

45. James the butler was offered a job for $8000 and a new car for a year of service. James quit after 7 months of service. He was given the car and $1600 for the 7 months of service. What was the dollar value of the car?

   (A) $6400   (B) $7200   (C) $7360   (D) $8000   (E) $15360

46. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\overset{\frown}{AED}$ given $m\angle BPC = 110\,°$ and $m\angle BAP = 30\,°$.
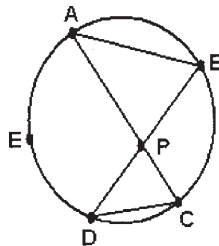

   (A) 160 °   (B) 150 °   (C) 140 °   (D) 110 °   (E) 80 °

47. Bear Foote High student body is made up of 60% boys and 40% girls. All of the boys wear sneakers. Some of the girls wear flip flops and some wear sneakers. A student is standing behind their locker and Principal Smokie can only see a pair of sneakers. What is the probability that the student is a girl?

(A) $33\frac{1}{3}\%$ (B) 30%   (C) 25%   (D) 20%   (E) $16\frac{2}{3}\%$

48. Which of the following equations will produce the graph shown here?
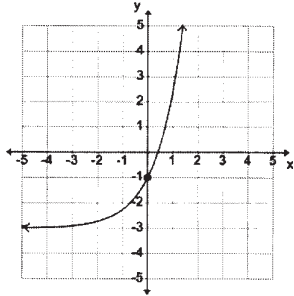
(A) $y = 2e^x - 3$ (B) $y = 2\ln(x) - 3$ (C) $y = \sqrt{e^x} - 3$ (D) $y = 2e^{(-x)} - 3$ (E) $y = \ln(x) - 3$

49. Cy Phy's physics class is trying to determine the magnitude of the sum if two forces, 220 and 180 lb, act at on an object at an angle of 46 °. What is the magnitude of their sum? (nearest lb).

(A) 89 lb   (B) 161 lb   (C) 223 lb   (D) 369 lb   (E) 381 lb

50. The type of graph of the polar equation $r = 1 + 2\theta$ is called a:

(A) Archimedian spiral   (B) cardiod   (C) lemniscate   (D) limacon   (E) rose

51. The polar graph of $r^2 = 4\sin(2\theta)$ is symmetric to the pole and to the:

(A) polar axis   (B) line $\theta = \pi$ (C) line $\theta = \frac{\pi}{2}$ (D) line $\theta = \frac{\pi}{3}$ (E) line $\theta = \frac{\pi}{4}$

52. The trapezoid shown exists such that BC = 10 cm, AD = 16 cm, $\overline{BC} \parallel \overline{EF} \parallel \overline{AD}$, and $\overline{EF}$ and the two diagonals intersect at a common point. Find EF. (nearest tenth)
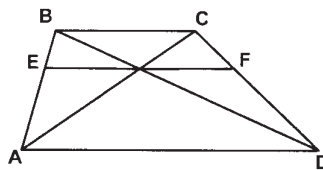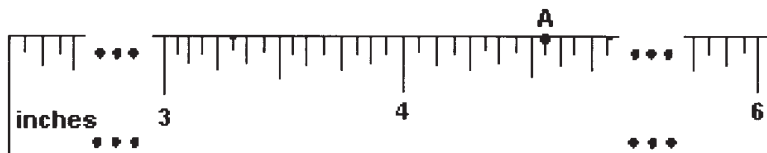

(A) 12.0 cm   (B) 12.3 cm   (C) 12.6 cm   (D) 13.0 cm   (E) 13.3 cm

53. Let $A = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$. Find $\det(A + A^{-1})$.

(A) -1 (B) -4 (C) -32 (D) -40 (E) -48

54. Bo Kutter uses a 6" ruler to measure the length of his ribbon. The ribbon has a length of A. He wants to cut the ribbon into four equal parts. How long will each part be?

(A) $\frac{25}{64}$ "   (B) $\frac{13}{16}$ "   (C) $1\frac{3}{32}$ "   (D) $1\frac{1}{8}$ "   (E) $1\frac{9}{64}$ "

55. I. C. Water skates around the edge of a circular pond at a linear velocity of 7.3 meters/second. The edge of the pond is 6 meters from the center of the pond. How many revolutions can I. C. make per minute?   (nearest tenth)

(A) 7.6 rpm   (B) 11.6 rpm   (C) 12.4 rpm   (D) 13.5 rpm   (E) 19.4 rpm

56. How many asymptotes exist of $h(x) = \frac{x^2 + 3x + 1}{4x^2 - 9}$?

(A) none exists   (B) 1   (C) 2   (D) 3   (E) 4

57. $\int [\sin(x)\cos(x)]\ dx = $ _____ $+ C$, where C is some arbitrary constant.

(A) $\cos^2(x) - 1$ (B) $\frac{1}{2}\cos^2(x)\sin(x)$ (C) $-\cos^2(x)\sin(x)$ (D) $-\sin(x)\cos(x)$ (E) $-\frac{1}{2}\cos^2(x)$

58. Wally World Discount Mart collected the following data about customer complaints.

Number of complaints in a day:	5	10	15	20
Probability of that number of complaints happening:	15%	30%	40%	15%

Based on this data, what is the expected number of complaints per day?

(A) 12   (B) 12.5   (C) 12.75   (D) 13   (E) 13.75

59. The number 2010 is a(n) _____ number.

(A) deficient   (B) happy   (C) lucky   (D) evil   (E) prime

60. One-centimeter cubes are glued together to form the object in the figure shown. Dee Z. Marker picks up the figure and puts a "Z" on all of the cubes' faces that can be marked on. How many of the cubes' faces will not be marked with a "Z"?
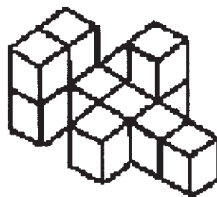
(A) 20   (B) 24   (C) 26   (D) 28   (E) 32

1. C	21. B	41. B
2. D	22. C	42. A
3. B	23. E	43. C
4. A	24. D	44. C
5. E	25. A	45. C
6. B	26. C	46. A
7. D	27. C	47. C
8. D	28. D	48. A
9. E	29. E	49. D
10. C	30. B	50. A
11. B	31. B	51. E
12. E	32. A	52. B
13. B	33. C	53. D
14. D	34. B	54. E
15. A	35. E	55. B
16. D	36. D	56. D
17. B	37. B	57. E
18. C	38. D	58. C
19. B	39. A	59. D
20. D	40. B	60. C