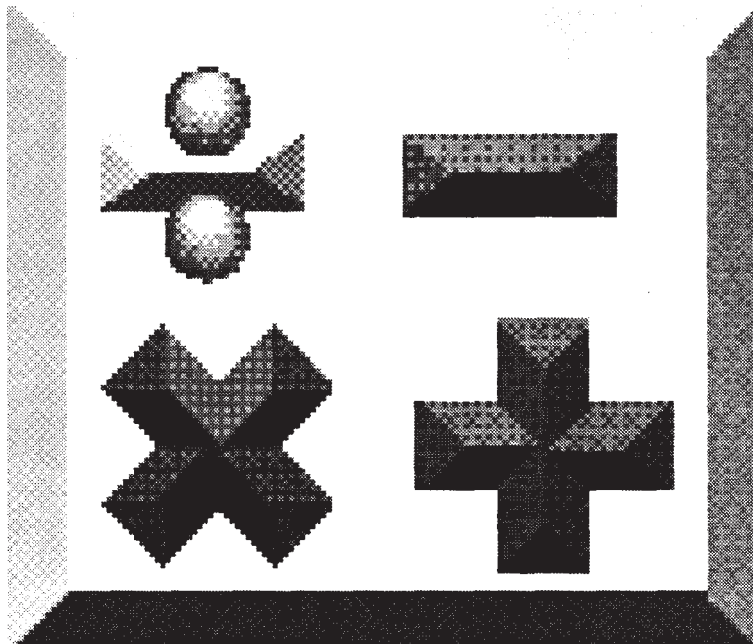UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics
District 1 • 2010

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $10 \times 8 + 6 \div (4 - 2) \div 3 - 5 \times 7 + 9$

   (A) 19.5   (B) 37   (C) 55   (D) $74\frac{1}{3}$ (E) 125

2. Bill Meelater is buying a set of 4 tires for his car. The tires cost \$49.50 each. There is a "buy 3 and get the 4th at half price" sale going on. If Bill pays with cash instead of credit card, he gets 8% off of his total bill before taxes. What is Bill's final cost before taxes if he pays with cash?

   (A) \$182.16   (B) \$161.00   (C) \$273.24   (D) \$173.17   (E) \$159.39

3. Which of the following is not a solution to $3|5 - 7x| < 9$?

   (A) 1   (B) $\frac{5}{7}$ (C) 1.1   (D) $\frac{1}{4}$ (E) 0.4

4. Ye Olde Nut Shoppe mixes pecans that cost \$1.85 per pound with some walnuts that cost \$2.45 per pound to make 20 pounds of mixed nuts that costs \$2.00 per pound. How many pounds of pecans are in the mixture?

   (A) 6 lb   (B) 10 lb   (C) 15 lb   (D) 9 lb   (E) 5 lb

5. Let $\triangle JKL$ be a right triangle where $\angle JKL = 90°$. Which of the following is the same point as K?

   (A) centroid   (B) circumcenter   (C) incenter   (D) orthocenter   (E) midcenter

6. A water tank in the shape of a rectangular prism contains approximately 718 gallons of water, which is 80% of the tanks maximum capacity. The length of the tanks base is 6 feet and its width is 5 feet. How tall is the tank? (nearest foot)

   (A) 3 ft   (B) 4 ft   (C) 5 ft   (D) 6 ft   (E) 7 ft

7. One-centimeter cubes are glued together to form the object in the figure shown. If the side of each cube has a length of 2 cm the two-dimensional perspective of the top view of this figure has a surface area of:
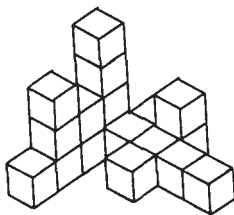
   (A) 11 cm^2 (B) 22 cm^2 (C) 36 cm^2 (D) 44 cm^2 (E) 60 cm^2

8. If you slice a complete cone (double cone) with a plane that is parallel to the axis of the cone, the intersection is a(n) _____ .

   (A) parabola   (B) ellipse   (C) circle   (D) hyperbola   (E) line

9. Determine the period of $y = 1 + 2\tan(3\pi(x) + 4)$.

   (A) $\frac{1}{4}$ (B) 3   (C) $\frac{1}{2}$ (D) 2   (E) $\frac{1}{3}$

10. How many elements are in $\left\{ x \mid 4\sin(x)\cos(x) = \sqrt{3}, x \in [0, \pi) \right\}$?

    (A) 1   (B) 2   (C) 4   (D) 6   (E) 8

11. Let $A = \begin{bmatrix} -2 & -3 \\ 5 & 8 \end{bmatrix}$. Find $\det(A + A^T)$.

    (A) -68 (B) -40 (C) -30 (D) -2 (E) -1

12. Les Dough received the following 5 year stock report. Based on the report. What was his average gain/loss over the 5 year period for the Sure2Make$$$ stock? (nearest tenth)

<u>Sure2Make$$$ Stock Report</u>

Year	1	2	3	4	5
Gain/Loss	+8%	+5%	−7%	−3%	+1%

    (A) $+0.3\%$ (B) $+0.7\%$ (C) $+0.8\%$ (D) $+1.0\%$ (E) $+1.1\%$

13. $\int \left(\frac{3}{x^2 - 2x + 1} \right) dx = \underline{\hspace{2cm}} + C$, where C is some arbitrary constant.

    (A) $6 \log(x - 1)$ (B) $\log(x - 1)^2$ (C) $-\frac{3}{x-1}$ (D) $\frac{(x-1)^2}{3}$ (E) $-\frac{6}{x-1}$

14. Find the slope of the line tangent to the curve $y = x^2 - 2x + 1$ at the point $(3, 4)$.

    (A) 6   (B) 4   (C) 2   (D) -1 (E) -3

15. Albe Darned lost the 3 number combination to the padlock shown. He knows that the first number is a prime number greater than 20, the second number is a composite number less than 20, and the third number is a perfect square greater than 10, but less than 39. How many combinations can he try to open the lock?

    (A) 60   (B) 90   (C) 99   (D) 120   (E) 132

16. Coach Newberry's math team is working on probability problems. His team consists of 4 boys and 2 girls. Coach randomly selects two students names without replacement. What is the probability that the first name drawn is a girl if it is known that the second will be a girl?

   (A) $\frac{1}{15}$ (B) $\frac{1}{14}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$

17. Which of the following mathematicians is best know for their work with 4-D POLYTOPES ?

   (A) Karen E. Smith   (B) George Boole   (C) Alicia Stott

   (D) Leonard Euler   (E) Freda Porter

18. The operation $\odot$ is defined as $x \odot y = y^2 - 2yx + x^2$, where x, y are integers and $x \neq y$. Find the value of $(2 \odot -3)(4 \odot -5)$.

   (A) 2025   (B) 436   (C) 106   (D) 1   (E) 0

19. Dee Zar uses a 6" ruler to measure the lengths of two pieces of string. One piece of string has a length of A and another piece of string has a length of B. What is the total length of the two pieces of string?
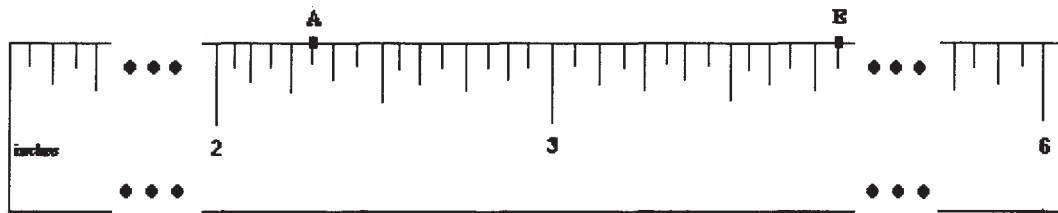
   (A) $1\frac{5}{8}$ "   (B) $6\frac{1}{8}$ "   (C) $5\frac{11}{16}$ "   (D) $6\frac{1}{4}$ "   (E) $1\frac{1}{2}$ "
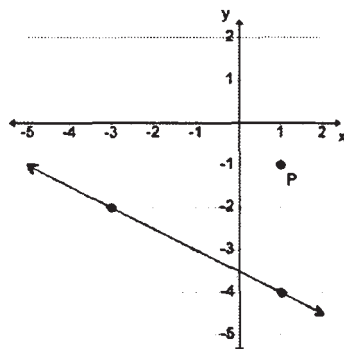
20. Which of the following is an example of the identity property of real numbers?

   (A) $a + 0 = a$ (B) $b = b$ (C) $c - c = 0$ (D) $d \times \frac{1}{d} = 1$ (E) $0 \div e = 0$

21. $111_2 + 111_3 + 111_4 = $ _____$_5$

   (A) 2313   (B) 40   (C) 111   (D) 333   (E) 131

22. Find an equation of a line through point P and parallel to the line shown.

   (A) $x + 2y = -1$ (B) $x - 2y = 3$ (C) $x - 2y = 1$ (D) $x + 2y = 3$ (E) $-x - 2y = -1$

23. The solution set of $4x + 7y = 56$ is $\{(x, y)| \ x,y \in \{\text{Integers}\} \text{ and } x,y \geq 0\}$. How many such ordered pairs exist?

   (A) 14   (B) 8   (C) 7   (D) 4   (E) 3

24. The points $(-1, 1)$ and $(2, -A)$ lie on the line $3x + By = -5$. Find A.

   (A) -0.5 (B) 1.375 (C) -0.125 (D) -5.5 (E) 2

25. A triangle with side lengths of 7 cm, 11 cm, and 11 cm is a(n) _____ triangle.

   (A) isosceles acute   (B) scalene obtuse   (C) isosceles obtuse   (D) scalene acute   (E) right

26. A circle with the center at C has a radius of 11 inches. A chord AB of the circle is 8 inches long. Find the distance, nearest quarter inch, from the chord to the center C.

   (A) $9\frac{1}{2}$ in   (B) $9\frac{3}{4}$ in   (C) $10\frac{1}{4}$ in   (D) $10\frac{3}{4}$ in   (E) $11\frac{1}{2}$ in
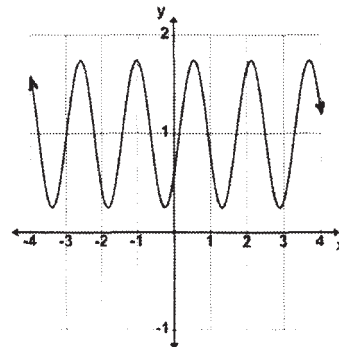
27. If $a_1 = -1$, $a_2 = 3$, $a_3 = -5$ and $a_n = a_{n-3} - a_{n-2} + a_{n-1}$, where $n \geq 4$, then a_7 equals:

   (A) -9 (B) 7   (C) -5 (D) 3   (E) -1

28. If $27^{(k)} = 9^{(2k+1)}$, then $3^{(3k+2)} = ?$

   (A) $\frac{1}{81}$ (B) 343   (C) $-\frac{1}{3}$ (D) -81 (E) 6561

29. The equation $y = $ _____ will produce this graph.

   (A) $\dfrac{3\sin(x+1) - 3\cos(x+1)}{4}$ (B) $\dfrac{4 - 3\cos(4x+1)}{4}$ (C) $\dfrac{3\cos(x+1) + 3\sin(x+1)}{4}$

   (D) $\dfrac{3 + 4\sin(3x+1)}{3}$ (E) $\dfrac{3 - 3\cos(4x-1)}{4}$

30. Which of the following is true about $f(x) = (x)^{-1}$?

   (A) It is an odd function.   (B) It is an even function.   (C) It has no asymptotes.

   (D) It is neither an even nor an odd function   (E) It is not one-to-one function.

31. Betty Lobs drops a tennis ball from a height of 12 meters. Each time it hits the ground it rebounds to a height of 60% of the distance it fell. Find the total distance the tennis ball travels when it reaches the ground the fourth time. (nearest tenth)

    (A) 13.2 m   (B) 26.1 m   (C) 38.1 m   (D) 40.2 m   (E) 52.2 m

32. Find the sum of the series $0.5 - \frac{(0.5)^2}{2} + \frac{(0.5)^3}{3} - \frac{(0.5)^4}{4} + \frac{(0.5)^5}{5} - ...$ (nearest thousandth)

    (A) 0.385   (B) 0.401   (C) 0.403   (D) 0.405   (E) 0.407

33. The eccentricity of the figure given by the equation $x^2 + 4y^2 - 6x - 16y = 11$ is:

    (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{3}$ (C) 2   (D) $\frac{1}{2}$ (E) $2\sqrt{3}$

34. The Millersview Gifted School has 2 administrators, 5 teachers, 10 boys, and 10 girls. An advisory committee containing 1 administrator, 2 teachers, 3 boys, and 4 girls is to be created. How many different committees can be created?

    (A) 342   (B) 15,000   (C) 504,000   (D) 288,000   (E) 270

35. Roland Bones tosses a pair of dice. What are the odds that he won't roll a 2, 3, 7, 11, or 12?

    (A) 3 to 1   (B) 2 to 5   (C) 2 to 1   (D) 5 to 7   (E) 1 to 3

36. The figure shown is rotated 180°. Then it is reflected over its negative diagonal. Finally it is reflected over its horizontal axis of symmetry. Which of the following figures is the result of these three transformations?

    (A)   (B)   (C)   (D)   (E)

37. C. N. Blurry glued 5 congruent cubes together to make this special figure. The total surface area of the figure is $11\frac{1}{4}$ sq. cm. What is the length of each cube's edge?

    (A) $\frac{1}{8}$ cm   (B) $\frac{3}{16}$ cm   (C) $\frac{1}{4}$ cm   (D) $\frac{3}{4}$ cm   (E) $\frac{7}{8}$ cm

38. Two billion two hundred thousand two plus three million three thousand three minus five thousand five hundred five results in the number X. How many zeros are in the number X?

    (A) 6   (B) 5   (C) 4   (D) 3   (E) 2

39. Mr. Lou Kuss folds the net shown into a cube. He adds the numbers on each pair of opposite sides. What is the largest sum Lou comes up with?
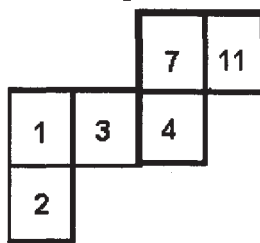

   (A) 14   (B) 13   (C) 11   (D) 9   (E) 6

40. Jan has 18 red marbles and 12 blue marbles. Dean has 20 marbles, all of them either red or blue. If the ratio of the red marbles to the blue marbles is the same for both Jan and Dean, then Jan has how many more red marbles than Dean?

   (A) 8   (B) 6   (C) 4   (D) 3   (E) 2

41. The coordinates of point A are $(0, -4)$. Point A is reflected across the line $y = -x$ to point B. Then point B is rotated 90° counterclockwise about the origin to point C. Then point C is translated vertically -4 units to point D. Find the coordinates of point D.

   (A) $(0, -8)$ (B) $(4, 4)$ (C) $(4, 0)$ (D) $(-4, 4)$ (E) $(0, 0)$

42. If two parallel lines are intersected by a transversal, then the exterior angles on the same side of the transversal are ____.

   (A) acute   (B) complementary   (C) congruent   (D) obtuse   (E) supplementary

43. Let $f(x) = x^3 - 1$ and $g(x) = x^2 - 4$ and $h(x) = x - 8$. Find $g\left(h\left(f(2)\right)\right)$.

   (A) 37   (B) -513 (C) 13   (D) -3 (E) -5

44. The graph of $x^2 + 0xy - y^2 + 0x + 0y + 0 = 0$ is a _____.

   (A) point (B) line (C) pair of intersecting lines (D) pair of parallel lines (E) hyperbola

45. Cy Figh points telescope A towards the northeast at an angle of elevation of 72° to see the Nova Star. He positions telescope B 2.5 miles due east of A and points it toward the northwest at an angle of elevation of 77° to see the same Nova Star. How far is the Nova Star from telescope A? (nearest tenth of a mile).

   (A) 4.3 mi   (B) 4.4 mi   (C) 4.5 mi   (D) 4.6 mi   (E) 4.7 mi

46. Which of the following is equivalent to $\sin(x) = \cos(\frac{x}{2})$?

   (A) $2\sin(x)\cos(x) = 0$ (B) $\sin^2(x) - 2\sin(x) + 1 = 0$ (C) $\sin(\frac{x}{2}) = 0$

   (D) $2\cos^2(x) + \cos(x) - 1 = 0$ (E) $\cos(2x) = 0$

47. If the three numbers 114, 170, and 233 are each divided by the number D, each of their quotients will have the same remainder R. Find R.

   (A) 2   (B) 3   (C) 5   (D) 7   (E) 9

48. Which of the following polar equations will produce this graph on a polar grid?
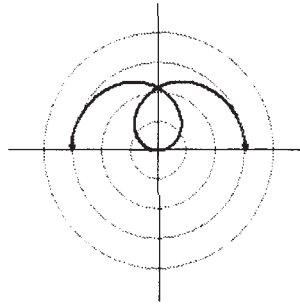
   (A) $r = 2\sin\left(\frac{\theta}{3}\right)$ (B) $r = 3\sin(2\theta)$ (C) $r = \frac{3\cos(\theta)}{2}$ (D) $r = 2\cos(3\theta)$ (E) $r = 3\cos\left(\frac{\theta}{2}\right)$

49. The instantaneous rate of change of the volume of a right-circular cone with a radius of 4 cm and an altitude twice its radius is K cm³/cm. Find K. (nearest tenth)

   (A) 100.5   (B) 33.5   (C) 16.8   (D) 50.3   (E) 301.6

50. The *Teechur Stor* sells red, blue, green, and black ink pens. They package the pens in sets of 3. How many different sets of 3 can they package?

   (A) 35   (B) 30   (C) 24   (D) 20   (E) 12

51. Suppose A, B, and C are positive integers such that $\frac{44}{9} = A + \cfrac{1}{B + \cfrac{1}{C}}$.
   The value of $\frac{1}{A} - \frac{1}{B} + \frac{1}{C}$ equals:

   (A) $-0.444...$ (B) -0.625 (C) -0.25 (D) 1.125   (E) 2.375

52. Which of the following is not a frugal number?

   (A) 125   (B) 128   (C) 243   (D) 256   (E) They are all frugal numbers

53. Herr Gauss has a box that contains the four letters of the word NINE. If he randomly selects one letter at a time and lines them up from left to right, what is the probability that the lined up letters will spell the word NEIN?

   (A) $\frac{1}{16}$ (B) $\frac{1}{12}$ (C) $\frac{1}{8}$ (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

54. In the binomial expansion of $(2x - 1)^5$, the coefficient of the x^2y^3 term is:

   (A) -10 (B) -20 (C) -40 (D) 20   (E) 10

55. Let $\|V_1\|=12$ and $\|V_2\|=18$. The angle between V_1 and V_2 is 60°. Find $\|V_1 + V_2\|$. (nearest tenth)

(A) 20.5   (B) 26.2   (C) 16.8   (D) 15.9   (E) 38.5

56. Willie Spendett will have saved $260.00 from his allowance by the end of this year. He plans to withdraw $\frac{1}{10}$ of his savings balance at the end of each week next year. At this rate, what will his balance be at the end of the 52nd week? (nearest cent)

(A) $1.09   (B) $1.26   (C) $1.34   (D) $2.60   (E) $3.86

57. $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, and $\overline{BC}$ are chords of circle O and intersect at point P. Find x, the measure of arc BC.
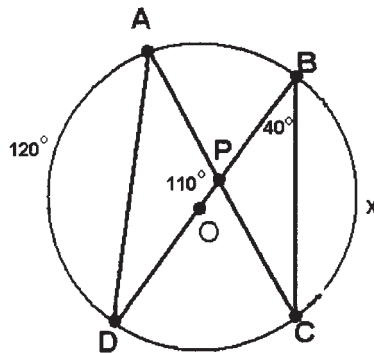
(A) 110°   (B) 100°   (C) 80°   (D) 75°   (E) 50°

58. If $18x^3 - 3x^2 - 6x = (ax)(ax - b)(bx + c)$ then $a + b + c =$ _____.

(A) 1   (B) 2   (C) 3   (D) 5   (E) 6

59. Simplify: $(a)^{-3} \times (ab)^{-2} \div (a^{-1}b^2)^{-1} \times (a^2b^3)^0$

(A) $a^{-4}b^4$ (B) a^4b^3 (C) $a^{-6}b^{-4}$ (D) $a^{-4}b^3$ (E) a^{-6}

60. Lorena hit the golf ball with her driver so that its initial speed was 180 feet per second. The ball left the club at an angle of 11°. Find the ball's initial speed in the vertical direction. (nearest foot)

(A) 177 ft/s   (B) 16 ft/s   (C) 101 ft/s   (D) 146 ft/s   (E) 34 ft/s

1. C	21. E	41. E
2. E	22. A	42. E
3. D	23. E	43. D
4. C	24. D	44. C
5. D	25. A	45. E
6. B	26. C	46. D
7. D	27. C	47. A
8. D	28. A	48. E
9. E	29. B	49. A
10. B	30. A	50. D
11. A	31. D	51. B
12. B	32. D	52. E
13. C	33. A	53. B
14. B	34. C	54. C
15. D	35. C	55. B
16. E	36. B	56. A
17. C	37. D	57. B
18. A	38. C	58. E
19. B	39. A	59. E
20. A	40. B	60. E