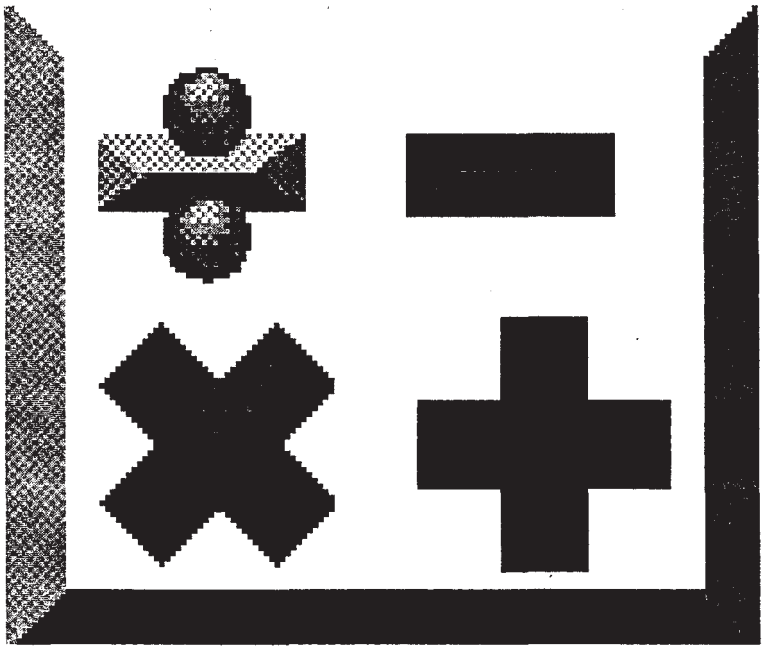
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2010



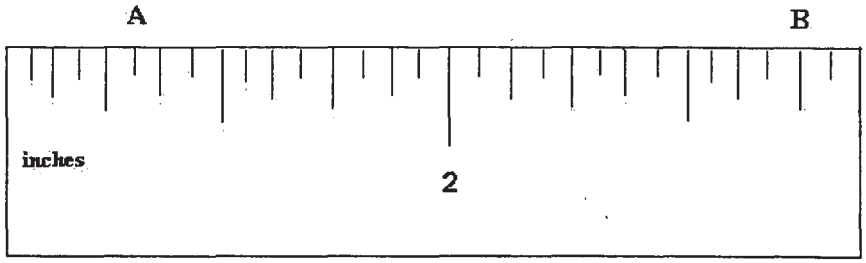
WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $\frac{7}{8} + \frac{3}{4} \div (\frac{5}{8} - \frac{1}{2}) \times \frac{3}{8} + \frac{1}{4} - \frac{1}{8}$

(A) $3\frac{1}{4}$ (B) $2\frac{45}{64}$ (C) $2\frac{9}{16}$ (D) $1\frac{3}{32}$ (E) $\frac{73}{80}$

2. Using the partial ruler shown below, find the distance from A to B.



(A) $1\frac{7}{8}$ " (B) $1\frac{1}{2}$ " (C) $1\frac{7}{16}$ " (D) $1\frac{9}{16}$ " (E) $1\frac{3}{4}$ "

3. If $ax + b = c$ and $c = dx + e$, then $ax + b = dx + e$ is an example of the _____ property.

(A) reflexive (B) associative (C) symmetric (D) distributive (E) transitive

4. May B. Fishy has a salt water aquarium. She mixes 5 gallons of water with some salt to make a 20% saline solution. The fish require a 16% solution. How much water will she have to add to make the required 16% saline solution?

(A) 200 oz (B) 160 oz (C) 128 oz (D) 120 oz (E) 96 oz

5. Find $f(5) + f(-1) + f(2)$ if $f(x) = \begin{cases} x - 3 & \text{if } x < 0 \\ 3x & \text{if } 0 < x < 3 \\ 3 - x & \text{if } x > 3 \end{cases}$

(A) -3 (B) 0 (C) 1 (D) 3 (E) 6

6. If $y = 1 - x$ and $y = \frac{2}{x}$ then $(x + y)(x^2 - xy + y^2) = ?$

(A) 7 (B) -2 (C) -5 (D) 8 (E) 3

7. Which of the following are the side lengths of a scalene acute triangle?

(A) 9, 40, 41 (B) 4, 7, 11 (C) 9, 10, 11 (D) 5, 5, 8 (E) 8, 7, 14

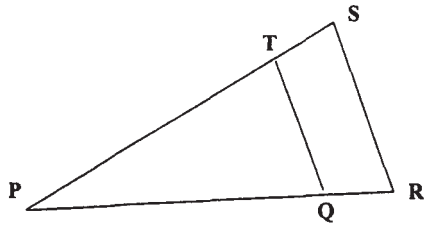
8. The point $(6, -6)$ is rotated 60 degrees clockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

(A) $(-6.7, -2.2)$ (B) $(-8.2, -2.2)$ (C) $(5.1, -8.2)$ (D) $(-8.2, -1.1)$ (E) $(8.2, 2.2)$

9. Find the quotient: $(x^4 + 2x^3 - 10x^2 + 22x - 15) \div (x^2 - 2x + 3)$

(A) $x^2 + 4x - 5$ (B) $x^2 + 5x - 6$ (C) $x^2 + 4x + 5$ (D) $x^2 - 4x + 3$ (E) $x^2 - 4x - 5$

10. In $\triangle PRS$, $QT \,//\, RS$, $RS = 4$, $QT = 3$, $ST = x$, and $PT = x + 5$. Find PS.



(A) 12 (B) 10 (C) 7.5 (D) 5 (E) not enough information given

11. The graph of $x^2 - 2xy + y^2 + 0x + 0y + 0 = 0$ is a _____.

(A) point (B) line (C) pair of lines (D) ellipse (E) hyperbola

12. Les Moolah has 28 coins. The coins are nickels and quarters and have a total value of \$ 4.00. How many more nickels than quarters does Les have?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

13. Which of the following is equivalent to $\frac{\sin\theta \tan\theta}{\sin(90° - \theta)} + \frac{\cot\theta}{\tan(90° - \theta)}$?

(A) $\sec^2\theta$ (B) $\cos\theta$ (C) $\tan^2\theta$ (D) $\sin\theta$ (E) 1

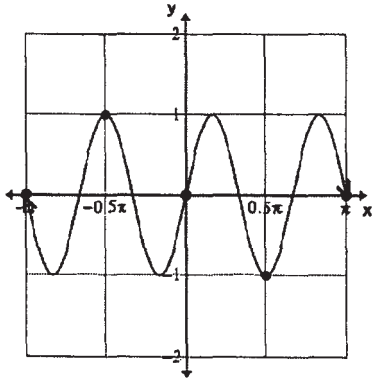
14. If $\cos x - \sin x = a$ and $\cos x + \sin x = b$, then $\cos^2 2x = ?$

(A) 2ab (B) $a - b$ (C) $a + b$ (D) $a^2 - 2ab + b^2$ (E) a^2b^2

15. Let $\|V_1\| = 9$, $\|V_2\| = 8$, where the direction angles of V_1 and V_2 are 60 ° and 150 °, respectively. Find the direction angle of $\|V_1 + V_2\|$. (nearest degree)

(A) 42 ° (B) 48 ° (C) 102 ° (D) 108 ° (E) 120 °

16. Which of the following is true about the relation graphed below?



(A) It is an odd function. (B) It is an even function. (C) It is not a function.

(D) It is neither an even nor an odd function (E) It is a one-to-one function.

17. Integers x & y exist such that $x = 2y$ and the arithmetic mean of x & y is 1 more than the harmonic mean of x & y. Find the geometric mean of x & y.

(A) $3\sqrt{2}$ (B) 9 (C) $2\sqrt{3}$ (D) 6 (E) $6\sqrt{2}$

18. Find k if $x + 4$ is a factor of $x^3 - x^2 + kx + 12$.

(A) 3 (B) -17 (C) 8 (D) -20 (E) 12

19. The focus of the figure given by the equation $x^2 + 6x - 12y + 57 = 0$ is (x , y). Find x.

(A) $(-3, 4)$ (B) $(0, 1)$ (C) $(-7, 4)$ (D) $(4, -3)$ (E) $(-3, 7)$

20. Let $f(x) = \frac{1}{x-1}$. Find the average rate of change of f(x) over the interval [2, 5].

(A) $-\frac{3}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{2}{7}$ (D) $1\frac{1}{3}$ (E) $3\frac{1}{2}$

21. Roland Bones found a die with 6 blank faces on it. He painted the numbers 1, 1, 2, 3, 5, & 8, one number per face, on the die. He created a game such that he gets 10 points if he rolls a composite number, he gets 5 points if he rolls a prime number, and he loses 7 points if he rolls a unit. What would the mathematical expectation be for any given roll?

(A) -7 pts (B) $-2\frac{1}{3}$ pts (C) 1 pt (D) $1\frac{5}{6}$ pts (E) 11 pts

22. Two distinct numbers are selected randomly from the set { 2 , 1 , 3 , 4 , 7, 11}. What is the probability that their sum is an odd number?

(A) $53\frac{1}{3}\%$ (B) $51\frac{1}{4}\%$ (C) 50% (D) $43\frac{3}{4}\%$ (E) $46\frac{2}{3}\%$

23. The figure shown is reflected over a negative diagonal. Which of the following figures is the result of that single transformation?

(A)  (B)  (C)  (D)  (E) 

24. A recent visit to the planet Strangebase discovered that the equation, $3S^2 - 25S + 66 = 0$, has two solutions, 4 and 9. What base was being used for the number system on planet Strangebase?

(A) base 5 (B) base 11 (C) base 13 (D) base 17 (E) base 36

25. On the map legend, 1 inch represents 120 miles. Beautiful downtown Millersview is 45 miles from San Angelo. How far is it on the map?

(A) $\frac{5}{16}$ " (B) $\frac{3}{8}$ " (C) $\frac{3}{4}$ " (D) $1\frac{1}{8}$ " (E) $2\frac{2}{3}$ "

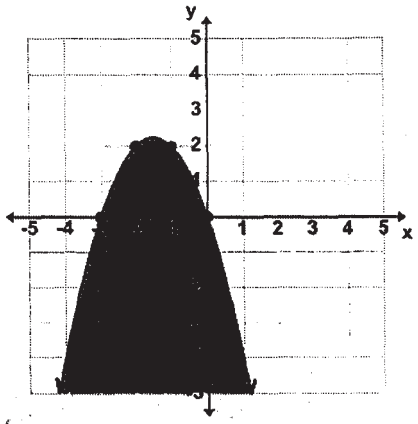
26. Which of the following is not a solution of $3 + 2|5x - 1| \leq 4$?

(A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{1}{6}$ (D) $\frac{2}{7}$ (E) $\frac{1}{8}$

27. If two parallel lines are intersected by a transversal, then the alternate interior angles are ____.

(A) acute (B) complementary (C) congruent (D) obtuse (E) supplementary

28. Which of the equations will produce the shaded portion of the graph shown?



(A) $y \leq -x^2 + 3x - 3$ (B) $y \leq -x^2 + 3x$ (C) $y \geq x^2 - 3x$

(D) $y \leq -(x^2 + 3x)$ (E) $y \geq -x^2 - 3x + 3$

29. Sir Vayor is trying to find the height of a flagpole. His eyes are 1.7 meters above the ground and he is standing 10 meters from the base of the pole. The angle of elevation from his eyes to the top of the pole is 60 °. Using this information Sir Vayor computes the top of the flagpole to be: (nearest meter)

(A) 10 m (B) 13 m (C) 15 m (D) 17 m (E) 19 m

30. Find the first term of the geometric sequence: a, b, 44, c, $19\frac{5}{9}$, ...

(A) $69\frac{19}{45}$ (B) $88\frac{2}{3}$ (C) 99 (D) 132 (E) $222\frac{3}{4}$

31. Evaluate: $\prod_{n=2}^{6} (1 + \frac{1}{n})$

(A) 1.45 (B) 3.5 (C) 6.1666... (D) 6.45 (E) 11.39

32. Coach Fuhrmann has 8 boys and 6 girls in his math and science club. He needs to send a delegation to a UIL planning conference. How many possible delegations can he send if each delegation must contain exactly 2 boys and exactly 2 girls?

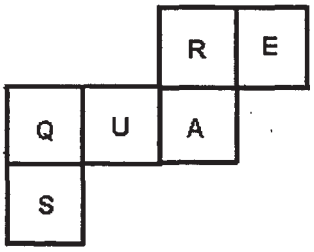
(A) 182 (B) 420 (C) 1,001 (D) 1,680 (E) 24,024

33. Which of the following mathematicians created an abacus for calculating products and quotients and extracting square roots that was based on Arab mathematics and lattice multiplication?

(A) John Venn (B) Sophie Germain (C) George Boole

(D) John Napier (E) Leonard Euler

34. Polly Euler folds the net shown into a cube. What letter will be on the opposite side of side S?



(A) Q (B) U (C) A (D) R (E) E

35. The Azusa Aztec band is selling band calendars to make money for their trip. They get 30% of the sales for the first 100 sold, 40% of the sales above 100 but less than or equal to 200, and 50% of the sales over 200. How much will the band make if they sell 275 calendars if each calendar sells for $10?

(A) $2062.5 (B) $1375.00 (C) $1100.00 (D) $1075.00 (E) $825.00

36. Simplify: $a^{-2} \times b^{2} \div a^{3} \div b^{-3} \times a \div b$

(A) $a^{-4}b^{4}$ (B) $a^{2}b^{0}$ (C) $a^{0}b^{-2}$ (D) $a^{4}b^{2}$ (E) $a^{-4}b^{-2}$

37. The points (2, 3) and (— 4, k) lie on the line $5x - 6y = C$. Find k.

(A) — 8 (B) — 3 (C) — 2 (D) 1 (E) 4

38. Les Quik, Moe Fass, and Willie Makit run in a 100 meter race. Les beat Moe by 10 meters and Moe beat Willie by 20 meters. If the runners ran at a constant speed, by how much did Les beat Willie?

(A) 8 meters (B) 10 meters (C) 18 meters (D) 28 meters (E) 30 meters

39. Point P(— 3 , 2) and point Q(4 , — 5) lie on the x-y plane. P is translated horizontally 2 units to the left. Q is reflected across the y-axis. What is the distance between the points after the translations? (nearest tenth of a unit)

(A) 6.1 (B) 7.1 (C) 7.6 (D) 9.5 (E) 11.4

40. If $a_1 = 2$, $a_2 = 3$, $a_3 = 5$ and $a_n = a_{n-1} + a_{n-2} - a_{n-3}$, where $n \geq 4$, then a_8 equals:

(A) 14 (B) 12 (C) 11 (D) 9 (E) 0

41. Find $f(g(1-x))$ when $f(x) = 3x - 1$ and $g(x) = x - 3$.

(A) $-3x-1$ (B) $5-3x$ (C) $-7-3x$ (D) $4x-4$ (E) $5-4x$

42. If $\sqrt{x\sqrt[3]{x\sqrt[4]{x}}} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 26 (B) 24 (C) 17 (D) 12 (E) 8

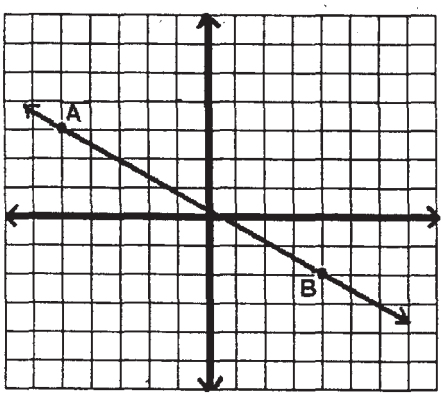
43. Using the equation $y = \frac{3}{4}\cos(2x - \frac{\pi}{3}) - 1$ which of the following has the largest numeric value?

(A) amplitude (B) displacement (C) frequency (D) period (E) phase shift

44. $\triangle ABC$ and $\triangle PQR$ exist such that $AB = BC = PQ = PR$, $m\angle ABC = 2x°$, $m\angle QPR = x°$, and they have equal areas. Find x.

(A) 15 (B) 30 (C) 45 (D) 60 (E) 75

45. Find an equation of a line parallel to line AB and passing through point $(-2, -3)$.



(A) $y = \frac{-5x-37}{9}$ (B) $y = \frac{9x-3}{5}$ (C) $y = \frac{-x-8}{2}$ (D) $y = 2x + 1$ (E) $y = \frac{5x-17}{9}$

46. A circle with the center at C has a radius of 9 cm. A chord AB of the circle is 6 cm long. Find the distance from the chord to the center C.

(A) $6\sqrt{2}$ cm (B) $3\sqrt{13}$ cm (C) $3\sqrt{10}$ cm (D) $3\sqrt{5}$ cm (E) $2\sqrt{14}$ cm

47. Find the determinant of the 3 x 3 matrix.

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

(A) -2 (B) -1 (C) 1 (D) 3 (E) 4

48. If $f'(x) = 15x^2 - 6x + 2$ and $f(-1) = -9$, find $f(1)$.

(A) 5 (B) 3 (C) -6 (D) 11 (E) -3

49. R_1, R_2 and R_3 are the roots of the equation $24x^3 + 26x^2 - 19x - 6 = 0$. R_1 and R_2 are the roots of the equation $12x^2 - 5x - 2 = 0$ as well. Find R_3.

(A) 2 (B) $\frac{2}{3}$ (C) $-\frac{1}{4}$ (D) $-1\frac{1}{2}$ (E) -4

50. $\int \sin(2x)\cos(2x)\ dx =$ _____ $+ C$, where C is an arbitrary constant.

(A) $-\frac{1}{4}\sin(2x)$ (B) $-2\sin(x)\cos(x)$ (C) $-\frac{1}{8}\cos(4x)$ (D) $-4\cos(4x)$ (E) $-\frac{1}{2}\sin(4x)$

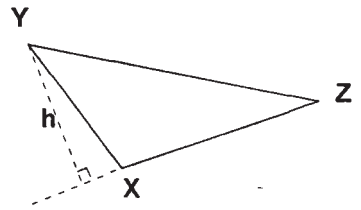
51. Willie Bettit has 5 plain red poker chips, 3 plain white poker chips, and 2 plain blue poker chips. How many ways can he line all of them up in a row?

(A) 1,440 (B) 3,628,800 (C) 120,960 (D) 5,040 (E) 2,520

52. Let $x = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$ be the continued fraction. Find x.

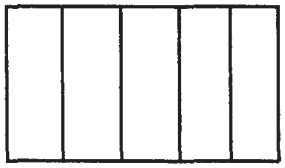
(A) $\frac{\sqrt{15}+1}{2}$ (B) $\frac{\sqrt{2}+1}{3}$ (C) $\frac{\sqrt{15}-3}{2}$ (D) $\frac{-\sqrt{15}-3}{2}$ (E) $\frac{\sqrt{2}-1}{3}$

53. Find the perimeter of $\triangle XYZ$ if XY = 8", XZ = 11" and $m\angle YXZ = 120°$. (nearest tenth)



(A) 22.7" (B) 35.5" (C) 33.4" (D) 32.6" (E) 30.4"

54. Elmoor Fudd is building a rectangular shaped pen for his porkie pigs. It will have 4 parallel fences dividing the pen into 5 sections as shown. If he has 600 feet of fencing, what is the maximum area of his pig pen?



(A) 6562.5 sq. ft. (B) 7000 sq. ft. (C) 7250 sq. ft. (D) 7500 sq. ft. (E) 8437.5 sq. ft.

55. How many 5 digit numbers can be made using the digits 1, 2, 3, 4 & 5 where the digits in the tens place and the hundreds place must be a prime number. Each digit can only be used once in a number.

(A) 18 (B) 24 (C) 36 (D) 42 (E) 48

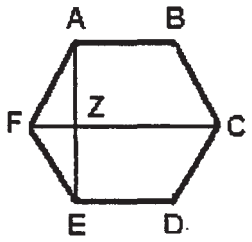
56. The Cowboys and the Texans will play twice this season. The Cowboys are twice as likely to win any game as the Texans. What is the probability that they will each win one of the two games?

(A) $33\frac{1}{3}$ % (B) $44\frac{4}{9}$ % (C) 50 % (D) $55\frac{5}{9}$ % (E) $66\frac{2}{3}$ %

57. Simplify: $\frac{(n+1)!}{(n-1)!} \div \frac{(n+2)!}{n!}$

(A) $\frac{n}{n+2}$ (B) n^2+2n (C) $n(n+1)(n+2)$ (D) $\frac{n+1}{n}$ (E) n^2-2n+1

58. Polygon ABCDEF is a regular hexagon and segments AE and CF intersect at point Z. The ratio of the area of triangle EFZ to the area of the quadrilateral ABCZ is:



(A) 1 : 2.5 (B) 1 : 3 (C) 1 : 4 (D) 1 : 5 (E) 1 : 6

59. The universal set $U = \{1,2,3,5,8,13,21,34\}$. Subset $A = \{1,3,8,21,34\}$ and subset $B = \{2,3,5,13,21\}$. How many elements are in the complement set of $A \cap B$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

60. Which of the following numbers is an unhappy and evil number?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

UIL HIGH SCHOOL SCIENCE CONTEST
ANSWER KEY

INVITATIONAL B • 2010

1.	C	21.	B	41.	C
2.	D	22.	E	42.	D
3.	D	23.	D	43.	D
4.	D	24.	A	44.	B
5.	D	25.	D	45.	E
6.	D	26.	C	46.	A
7.	B	27.	E	47.	C
8.	C	28.	D	48.	C
9.	B	29.	B	49.	E
10.	B	30.	E	50.	C
11.	E	31.	E	51.	B
12.	D	32.	D	52.	A
13.	A	33.	B	53.	A
14.	A	34.	C	54.	B
15.	B	35.	B	55.	E
16.	E	36.	C	56.	E
17.	A	37.	C	57.	D
18.	C	38.	B	58.	B
19.	A	39.	D	59.	E
20.	D	40.	D	60.	C