UiL

UNIVERSITY INTERSCHOLASTIC LEAGUE
Making a World of Difference

Mathematics
Regional • 2009

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sqrt{49} = 7$$

**WRITE ALL ANSWERS WITH
CAPITAL LETTERS**

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $0.4 - \frac{3}{5} \times 0.3 + \frac{1}{5} \div 0.1$

   (A) $\frac{21}{50}$ (B) $\frac{3}{5}$ (C) $1\frac{1}{5}$ (D) $1\frac{2}{5}$ (E) $2\frac{11}{50}$

2. $4a \times (4a)^{-1} = 1$ and $-4a + 4a = 0$ are examples of the _____ property of equality.

   (A) associative   (B) commutative (C) distributive (D) identity   (E) inverse

3. Simplify: $\left(\frac{4x^2 - 1}{2x^2 + 3x - 2}\right)\left(\frac{x^2 + 4x + 4}{4x^3 + 8x^2 - x - 2}\right)$

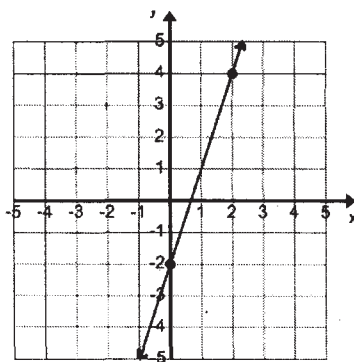   (A) $\frac{1}{2x-1}$ (B) $\frac{1}{x+2}$ (C) $2x^2 + 5x + 2$ (D) $3x + 5$ (E) $x^2 + 4x + 4$

4. Find an equation of the line shown using the two points shown.

   (A) $3x - 2y = 4$ (B) $4x - 3y = 10$ (C) $3y - x = 2$ (D) $3x - y = 2$ (E) $3y - 2x = 6$

5. A ray with an endpoint on the exterior of a circle extends through the circle intersecting it at two distinct points. The ray is called a/an _____.
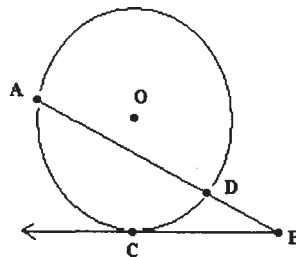
   (A) chord   (B) diameter   (C) radius   (D) secant   (E) tangent

6. Using the figure below, if $m\angle ABC = 20°$ and $m\overset{\frown}{AC} = 70°$ then $m\overset{\frown}{CD} =$ _____.

   (A) $50°$ (B) $45°$ (C) $35°$ (D) $30°$ (E) $20°$

7. Let $f(x) = 2x - 5$ and $g(x) = x + 3$. Find $f^{-1}\left(g\left(f(-x)\right)\right)$.

   (A) $1.5 - x$ (B) $-4x - 9$ (C) $x - 3$ (D) $4 - x$ (E) $-2x - 4$

8. If $a_1 = -1$, $a_2 = -2$, $a_3 = -3$ and $a_n = (a_{n-1})(a_{n-3}) + (a_{n-2})$, where $n \geq 4$, then $a_7 = \underline{\quad}$.

   (A) -16 (B) -5 (C) 1 (D) 11 (E) 39

9. A vulture is perched on top of a telephone pole that is 30 feet tall. Looking down at an angle of depression of 35° he sees some "road kill" on the highway. How far from the base of the pole is the "road kill"? (nearest foot)

   (A) 21 ft.   (B) 25 ft.   (C) 34 ft.   (D) 43 ft.   (E) 64 ft.

10. Let $f(x) = \frac{1}{2} + \frac{3}{2} \sin(4x + \frac{4\pi}{3})$. Which of the following has a value of $\frac{\pi}{3}$?

   (A) amplitude   (B) displacement (vertical)   (C) frequency   (D) phase shift   (E) period

11. Find the determinant: $\begin{bmatrix} -1 & 2 & -3 \\ 1 & -2 & 3 \\ -1 & 2 & -3 \end{bmatrix}$

   (A) -6 (B) -3 (C) -2 (D) -1 (E) 0

12. $\displaystyle\prod_{k=1}^{3} (kx - k + 1) = ?$

   (A) $6x^3 - 7x^2 + 2x$ (B) $6x - 3$ (C) $6x^2 - 7x + 2$

   (D) $6x^3 - 19x^2 + 16x - 4$ (E) $6x^3 + 5x^2 - 12x + 4$

13. A point of inflection on the graph of $f(x) = -x^3 - 3x^2 - 3x - 10$ is (a, b). Find b.

   (A) 3 (B) 1 (C) 0 (D) -1 (E) -9

14. Using the inequality $\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos(x)}{x^2} < \frac{1}{2}$, the value of $\displaystyle\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}$ is ?

   (A) $\frac{25}{52}$ (B) $\frac{5}{12}$ (C) $\frac{11}{24}$ (D) $\frac{1}{2}$ (E) 1

15. Les Sense has 3 identical pennies and 3 identical dimes. He wants to place a single coin in each of the circles shown below. How many different ways can Les place the coins if rotation doesn't matter?
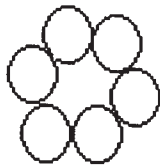

   (A) 20   (B) 12   (C) 10   (D) 6   (E) 4

16. Three fair six-sided dice are tossed and the spots on the top faces are recorded. One die is red, one is white, and one is blue. The probability of one of the following is 25%. Which one is it?

    (A) All three numbers are even.
    (B) The red die is a six or the blue die is even.
    (C) Exactly two of the three numbers are equal.
    (D) The red die is even and the blue die is 4 or more.
    (E) None of the above

17. Let AB be the diameter of the circle with center C with $CG \perp AB$, $DE \perp AB$, and $EF \perp DC$. Let AE = 9, BE = 5, and CG = 2. Find DE. (nearest hundredth)
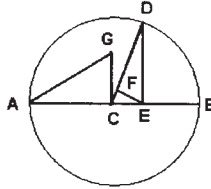
    (A) 6.71   (B) 7.28   (C) 6.43   (D) 6.25   (E) 7.00

18. Which of the following mathematicians' major contributions included a system of latitude and longitude, a measurement of the Earth's circumference, and a prime number sieve?

    (A) Emmy Noether (B) Euclid   (C) Euler   (D) Eratosthenes (E) Emilie du Châtelet

19. Melody Toones wants to buy 5 music CDs. The cost of a single CD is $20.00. The cost of a 2-pack set of CDs is 20% less than the cost of the 2 single CDs. The cost of the 3-pack set is 40% less than the cost of 3 single CDs. How much would Melody save if she buys the sets instead of the singles?

    (A) $16.00   (B) $20.00   (C) $25.00   (D) $32.00   (E) $44.00

20. $4444_5 + 333_4 + 22_3 + 1_2 = $ _____ $_6$.

   (A) 116   (B) 1234   (C) 3120   (D) 5432   (E) 55555

21. Five pounds of regular blend coffee sells for $6.50 per pound. Three pounds of super blend coffee sells for $8.00 per pound. Find the cost per pound of the special mixture of the regular blend and the super blend. (nearest cent)

    (A) $7.06   (B) $7.41   (C) $6.85   (D) $7.00   (E) $7.25

22. The equation $4x^2 - 3x - k = 0$ always has two positive roots when which of these is true?

    (A) $-1 < k < 0$ (B) $-\frac{9}{16} > k > -1$ (C) $k < -1$ (D) $k > -\frac{9}{16}$ (E) $-\frac{9}{16} < k < 0$

23. **Find the volume of the cone. (nearest tenth)**
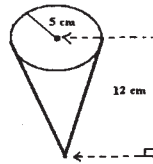
(A) 471.2 cm^3 (B) 188.5 cm^3 (C) 314.2 cm^3 (D) 235.6 cm^3 (E) 340.3 cm^3

24. **The coordinates of the vertices of $\triangle ABC$ are $(-2, -1)$, $(1, 2)$ and $(4, -1)$. The coordinates, nearest hundredth, of the incenter is:**

(A) $(2.34, 0.88)$ (B) $(1.00, 0.24)$ (C) $(1.00, -0.76)$ (D) $(2.33, 1.33)$ (E) $(1.00, 0.00)$

25. **If $p + q = 11$ and $p \times q = 22$ then $(p - q)^2 = ?$**

(A) 11   (B) 33   (C) 121   (D) 209   (E) 242

26. **Let $\left[\log_a(18k) - \log_a(2)\right] + \log_a(4k) = 2\log_a(x)$. Find x.**

(A) 13k   (B) 11k − 1   (C) 6k   (D) 18k   (E) $36k^2$

27. **The annual Lake Ivie Jetski Race starts at marker X, goes due North to marker Y, a distance of 500 yards. At marker Y, the racers travel on a bearing of 250° to marker Z, a distance of 750 yards. At marker Z they race straight back to marker X. Find the bearing they must take get from marker Z to marker X. (nearest degree)**

(A) 120°   (B) 109°   (C) 78°   (D) 39°   (E) 20°

28. **The expression $\frac{\sin 2A}{1 - \cos 2A}$ is equivalent to which of the following?**

(A) $\cot 2A$ (B) $\tan A$ (C) $\tan(\frac{\pi}{2} - A)$ (D) $\cot(\frac{\pi}{2} + A)$ (E) $\frac{\tan 2A}{1 - \cot 2A}$

29. **U. R. Kwik can make 10 ice cream sundaes in 5 minutes. I. M. Kwiker can make 10 sundaes in 3 minutes. Soh Slo can make 10 sundaes in 10 minutes. How long would it take all three of them to make 10 sundaes in they worked together? (nearest second)**

(A) 1 min 40 sec   (B) 3 min 8 sec   (C) 36 sec   (D) 1 min 35 sec   (E) 1 min 18 sec

30. **An infinite geometric sequence has a common ratio of $-\frac{1}{2}$ and a sum of $-\frac{4}{9}$. What is the first term of the sequence?**

(A) $\frac{1}{3}$ (B) $\frac{2}{9}$ (C) $-\frac{2}{3}$ (D) $-\frac{9}{8}$ (E) $-\frac{2}{9}$

31. **The curve of a polar equation $r = a + b \cos \theta$ is called a looped limacon when which of the following is true:**

(A) $a = b$ (B) $a < b < 2a$ (C) $a < b$ (D) $2a \le b$ (E) $a > b$

32. Let $f'(x) = 9x^2 + 4x - 2$ and $f(-1) = 4$. Find $f(1)$.

    (A) -1 (B) 0   (C) 1   (D) 4   (E) 6

33. The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many distinct combinations exist if P is a factor of 35, Q is a perfect cube, and R is a Fibonacci number? Stopping at 0 is not allowed.
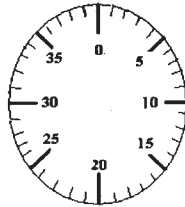
    (A) 108   (B) 96   (C) 64   (D) 48   (E) 15

34. How many subsets containing at least one member, but no more than 4 members, can be made from the set {♣,♥,⊕,♦,★}?

    (A) 30   (B) 24   (C) 2500   (D) 100   (E) 10

35. Let P = { T, M, S, C, A }, Q = { T, E, X, A, S}, and R = { M, A, T, H }. How many elements are in $(P \cap Q) \cup (P \cap R)$?

    (A) 2   (B) 4   (C) 5   (D) 6   (E) 8

36. Which of the following is NOT a solution to $1 + 2|3x - 4| \leq 5$?

    (A) $\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$ (E) $\frac{1}{5}$

37. Point J is on the negative y-axis. It is rotated 90° counter clockwise to point K. Point K is reflected across the y-axis to point L. Point L is translated vertically up to point M. Point M is reflected across the origin to point N. Where is point N?

   (A) y-axis   (B) QI   (C) QII   (D) QIII   (E) QIV

38. Which of the following is a reference angle for 4.8π ?

    (A) $\frac{3\pi}{10}$ (B) 0.7π (C) $\frac{4\pi}{5}$ (D) 1.2π (E) $\frac{\pi}{5}$

39. Which of the following is true about the relation $f(x) = x^3 - 5x - 1$?

    (A) odd function   (B) even function   (C) neither even nor odd function

    (D) not a function   (E) none of these are true

40. Which of the following would best represent a two dimensional perspective of the right rear view of this figure shown?

left
rear

right
rear
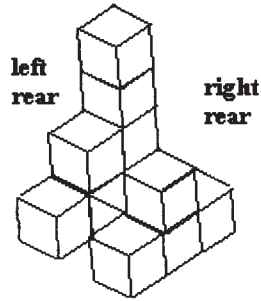
(A)

(B)

(C)

(D)

(E)

41. The function $f(x) = \frac{3}{2-x} - 5x$ is increasing at which of the following values of x?

(A) 2.4

(B) -1.4

(C) -0.4

(D) 4.4

(E) 3.4

42. Roland Bones rolls three fair six-sided dice, one die at a time. What is the probability that the he rolls a composite number on the first die, a prime number on the next die, and a unit on the last die?

(A) $\frac{1}{216}$

(B) $\frac{1}{6}$

(C) $\frac{1}{36}$

(D) $\frac{1}{72}$

(E) $\frac{1}{3}$

43. Find the area of the bases of this prism. All angles are right angles.

3

5

2

6

6

6

2
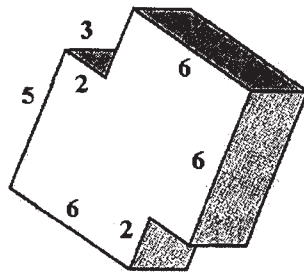
(A) 90 units ^{2}

(B) 108 units ^{2}

(C) 144 units ^{2}

(D) 162 units ^{2}

(E) 198 units ^{2}

44. Find the area of the region bounded by the graphs of $y = \ln(x+1)$, $y = 1 - x$, and $y = 0$. (closest approximation)

(A) 0.231 units^2

(B) 0.226 units^2

(C) 0.224 units^2

(D) 0.185 units^2

(E) 0.146 units^2

45. Let $f(x) = \frac{2x^2}{1-x}$ and s(x) be the slant asymptote of f. Find the slope of s(x).

(A) -2

(B) -1

(C) -0.5

(D) 1

(E) 2

46. Use the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, to transform the conic $3xy - 1 = 0$ into an equation that is in standard position and does not contain an xy term. The transformed equation is:

   (A) $x^2 - y^2 = 3$ (B) $3x^2 + 2y^2 = 1$ (C) $x^2 - 3y^2 = 2$ (D) $3x^2 + 3y^2 = 1$ (E) $3x^2 - 3y^2 = 2$

47. Phil Upp can fill his circular wading pool in 45 minutes using two hoses. He can fill the pool in 1 hour 20 minutes using the larger hose by itself. How long would it take the him to fill the pool if he uses the smaller hose by itself? (nearest minute)

   (A) 1 hr 50 min   (B) 1 hr 43 min   (C) 1 hr 35 min   (D) 1 hr 30 min   (E) 1 hr 28 min

48. The depth d of water at Rod Bending's favorite fishing hole in the Gulf varies sinusoidally from 5 feet to 15 feet, depending on the time t. Last Saturday, the high tide occurred at 5:00 a.m. and the next high tide occurred at 5:00 p.m. Which of the following equations best describes the relationship between the depth of the water and the time?

   (A) $d = 10 + 5\sin(\frac{\pi}{6}t - \frac{\pi}{3})$ (B) $d = 10 + 5\sin(\frac{\pi}{6}t - 2)$ (C) $d = 10 - 5\sin(\frac{\pi}{6}(t + \frac{1}{2}))$

   (D) $d = 10 - 5\sin(\frac{\pi}{3}t)$ (E) $d = 10 + 5\sin(\frac{\pi}{6}t + \frac{\pi}{3})$

49. How many points of intersection occur when $r = 2\sin(\theta) - 3$ and $\theta = \frac{3\pi}{2}$ are graphed on a polar coordinate system?

   (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

50. Joy Ryder goes on a roller coaster ride at the state fair. The graph best depicts the 6 minute ride. (speed is not truly linear in this case). During the time interval of 3 minutes to 4 minutes Joy's roller coaster car is _____.
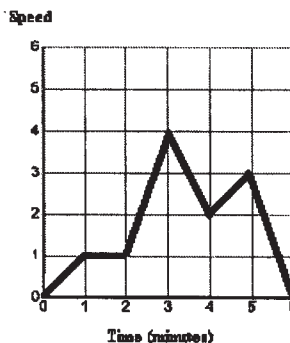
   (A) climbing uphill   (B) turning on a curve   (C) going downhill

   (D) decreasing speed   (E) increasing speed

51. The Fibonacci numbers (denoted by F) are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ... .
   The Lucas numbers (denoted by L) are 2, 1, 3, 4, 7, ... Find L_{12}.

   (A) 521   (B) 377   (C) 322   (D) 233   (E) 199

52. A rectangular shaped prism water tank has a maximum capacity of 360 gallons. The tank is 21 feet long and 11 inches deep. How wide is the tank?

    (A) 30"   (B) 31.5"   (C) 33"   (D) 34.5"   (E) 36"

53. The vertices of a pentagon are (2,0), (0,1), (0,3), $(-2, 1)$, and $(0, -2)$. What is the area of the pentagon formed by these points?

    (A) 5.5 units^2 (B) 6 units^2 (C) $6\sqrt{2}$ units^2 (D) $7\sqrt{2}$ units^2 (E) 8 units^2

54. Rob Deebank has a bag of $1.00 bills, $5.00 bills, $10.00 bills, $20.00 bills, $50.00 bills, and $100.00 bills. He hands out 4 bills to people who walk by him and smile. How many different sets of 4 bills could a smiling person receive?

    (A) 7560   (B) 100   (C) 330   (D) 744   (E) 126

55. Find the mean of the median, mode, and range of 8, 6, 7, 8, 9, 4, 7, 6, 6, 5, 9, & 5

    (A) 5   (B) $5\frac{5}{6}$ (C) 6   (D) $6\frac{1}{2}$ (E) $8\frac{3}{4}$

56. Juan Moorwin has lost 30% of the 40 games he has bowled. If he bowls 40 more games how many of them must he win to raise his overall winning percentage to 80%?

    (A) 36   (B) 32   (C) 28   (D) 20   (E) 18

57. How many ordered pairs (x , y) are solutions to the equation $4x + 3y < 10$, where x,y are integers and $x \geq 0$ and $y \geq 0$?

    (A) 3   (B) 4   (C) 7   (D) 10   (E) 12

58. Les Kwik took a leisurely drive in the country. He drove 2 hours at a constant speed before stopping for a picnic. After the picnic he drove 3 hours at a constant rate 15 mph faster than he drove before the picnic. If the total distance he traveled was 250 miles, what was his speed before the picnic?

    (A) 56 mph   (B) 35 mph   (C) 48 mph   (D) 50 mph   (E) 41 mph

59. Antonio Pottier is organizing his math students into teams of 5. He has 8 seniors, 6 juniors, 4 sophomores, and 2 freshmen. How many different teams consisting of 2 seniors, 1 junior, 1 sophomore, and 1 freshman could be created?

    (A) 4845   (B) 40   (C) 225   (D) 1344   (E) 384

60. Find the arithmetic mean of the roots of $x^3 - 7x^2 + 14x - 8 = 0$.

    (A) $1\frac{5}{7}$ (B) $2\frac{3}{10}$ (C) $2\frac{1}{3}$ (D) $2\frac{2}{3}$ (E) $4\frac{2}{3}$

1. E	21. A	41. A
2. E	22. E	42. C
3. A	23. C	43. B
4. D	24. B	44. A
5. D	25. B	45. A
6. D	26. C	46. E
7. A	27. B	47. B
8. D	28. C	48. A
9. D	29. D	49. C
10. D	30. C	50. D
11. E	31. C	51. C
12. A	32. E	52. A
13. E	33. B	53. E
14. D	34. A	54. E
15. E	35. B	55. B
16. D	36. E	56. A
17. A	37. E	57. C
18. D	38. E	58. E
19. D	39. C	59. D
20. C	40. D	60. C