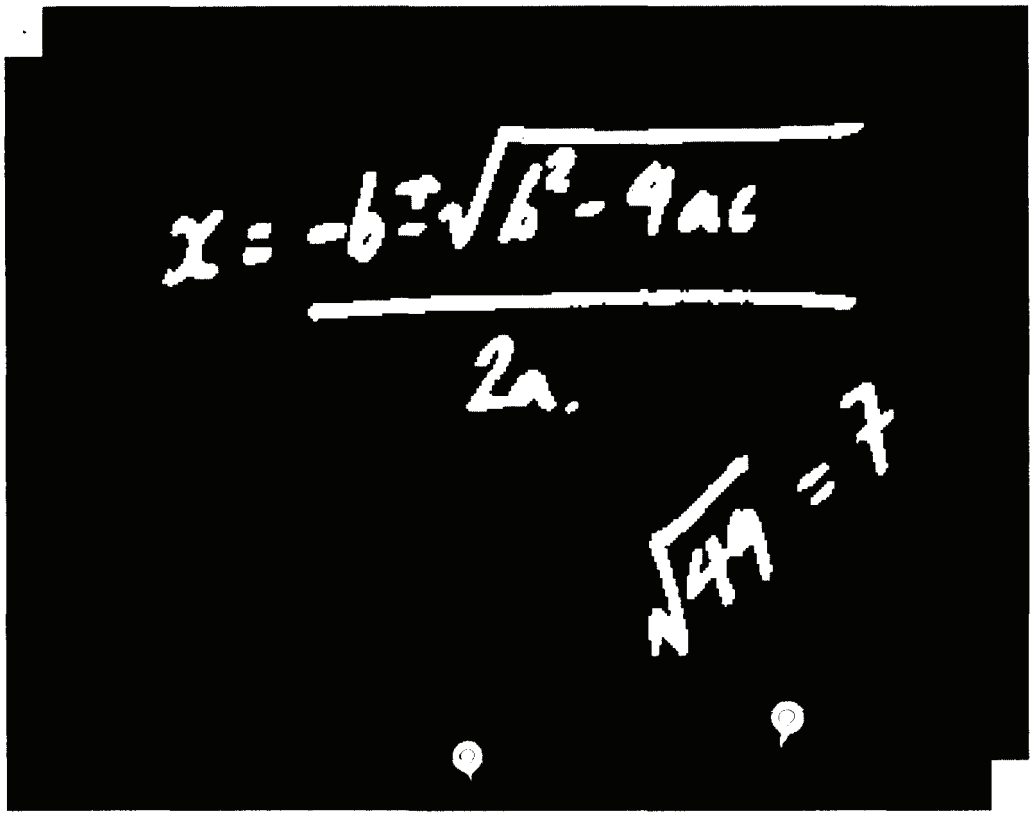
UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

District 2 • 2009



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

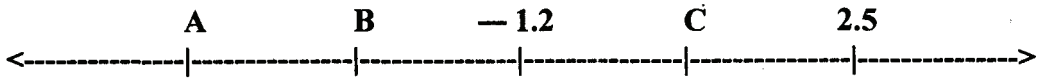
1. Evaluate: $6 \times 5^2 \div (4-1)! + 2^0 - 3$

(A) 148 (B) 8 (C) 42 (D) 23 (E) 33.5

2. The Bonnet Shoppe is having an Easter Bonnet sale. The regular price of their bonnets is $24.50. The sale price is $18.50. Senior women over the age of 60 get a coupon for 20% off the sale price. What percent of the regular price do the senior women pay? (nearest whole percent)

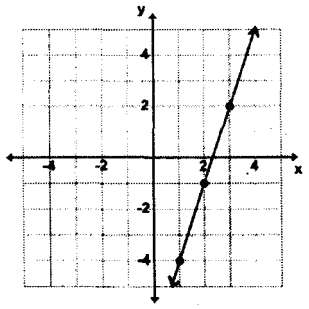
(A) 60% (B) 56% (C) 55% (D) 45% (E) 40%

3. The distances between the hash marks (|) are equal. Find A + B + C.



(A) — 8.6 (B) — 7.3 (C) — 4.35 (D) 1.85 (E) 5.55

4. A line through the origin is perpendicular to the line shown. The point of intersection is (x, y). Find x + y.



(A) 2.8 (B) 1.05 (C) 1 (D) 0.525 (E) 1.4

5. If the roots of $x^3 + bx^2 + cx + d = 0$ are — 3, — 4, and 2, then $b + c + d$ equals:

(A) 31 (B) 27 (C) 21 (D) — 5 (E) — 21

6. Rowan Boatright took 4 hours to travel upstream against a 3 mph current in his motor boat. It took 2.5 hours to return back to where he started. How far did Rowan travel?

(A) 26 miles (B) 40 miles (C) 42.25 miles (D) 80 miles (E) 84.5 miles

7. Point P is on the negative x-axis. It is translated vertically up to point Q. Point Q is reflected across the y-axis to point R. Point R is rotated $\frac{3\pi}{2}$ radians about the origin to point S. Point S is reflected across the x-axis to point T. Where is point T?

(A) y-axis (B) QI (C) QII (D) QIII (E) QIV

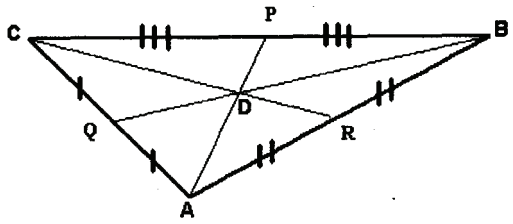
8. Let $p^2x^2 - q^2y^2 = 0$, where $p > 0$ and $q < 0$. Which of the following would be the best graphical representation of this equation?

(A) point (B) line (C) parallel lines (D) intersecting lines (E) no graph

9. Two chords, WX and YZ are in the same circle and do not intersect. If WX > YZ then the measure of $\overset{\frown}{WX}$ is _____ the measure of $\overset{\frown}{YZ}$.

(A) less than (B) one-half (C) equal to (D) double (E) greater than

10. If AD = 12 cm then DP = ?

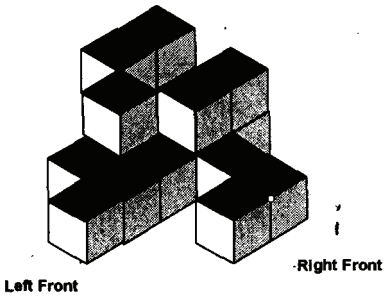


(A) 18 cm (B) 12 cm (C) 9 cm (D) 6 cm (E) 4 cm

11. Simplify: $\frac{a^2}{b^{-2}} \times \frac{b^3}{a^{-3}} \div (ab)^{-4}$

(A) $(ab)^9$ (B) $a^{-2}b^{-2}$ (C) a^9b^2 (D) a^2b^2 (E) ab

12. One-centimeter cubes are glued together to form the object in the figure shown. The two-dimensional perspective of the left front view of this figure has a perimeter of:



(A) 10 cm (B) 12 cm (C) 14 cm (D) 16 cm (E) 18 cm

13. The Stormchaser leaves port and sails 30 nautical miles due east. Then she changes direction and sails 30 nautical miles on a bearing of 120°. What bearing will she have to travel to sail straight back to port? (nearest degree)

(A) 75° (B) 300° (C) 15° (D) 285° (E) 330°

14. How many points of intersection occur when $r = \sin(3\theta)$ and $\theta = -\frac{\pi}{2}$ are graphed on a polar coordinate system?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. The sum of the coefficients of the 2nd term in the expansion of $(x - 2y)^4$ and the 4th term in the expansion of $(2x - y)^4$ is:

(A) −16 (B) −12 (C) 0 (D) 8 (E) 24

16. The sales at Honest Abe's car lot fluctuated from a high of $40,000 in January to a $25,000 in July. Which of the following equations best models the monthly sales at Honest Abe's car lot?

(A) $s(t) = 15\cos(\frac{\pi}{6}t) + 12.5$ (B) $s(t) = 7.5\cos(\frac{\pi}{6}t) + 25$ (C) $s(t) = 7.5\cos(\frac{\pi}{6}t) + 32.5$

(D) $s(t) = 15\cos(\frac{\pi}{12}t) + 32.5$ (E) $s(t) = 7.5\cos(\frac{\pi}{12}t) + 20$

17. Al G. Brah rode his bicycle from his camp to the end of the bike trail at an average speed of 3.5 mph and returned to his camp on the same bike trail at an average speed of 3.0 mph. What was Al's average speed for the round trip? (nearest thousandth)

(A) 3.225 mph (B) 3.231 mph (C) 3.240 mph (D) 3.250 mph (E) 3.260 mph

18. If $5^{(x+y)} = 625$ and $5^{(x-y)} = \frac{1}{5}$, then $5^{(x^2-y^2)} = ?$

(A) 5 (B) $\frac{1}{125}$ (C) $\frac{1}{625}$ (D) $\frac{1}{25}$ (E) 4

19. Minnie Mumm has a rectangular sheet of cardboard that is 3 feet by 4 feet. She is going to cut out the same size square from each of the four corners, then fold up the sides and make a box with maximum volume. What will the area of the bottom of the box be? (nearest square inch)

(A) 437 sq. in. (B) 450 sq. in. (C) 772 sq. in. (D) 795 sq. in. (E) 5240 sq. in.

20. Let k be a number in the domain of the function f. If $f'(k) = 0$ or $f'(k)$ does not exist, then k is called a(n) _____ point of function f.

(A) increasing (B) inflection (C) tangent (D) critical (E) decreasing

21. The curve of a polar equation $r = 1 + k\cos\theta$, where $k \geq 2$, is called a:

(A) circle (B) cardioid (C) spiral (D) lemniscate (E) limacon

22. Evaluate: $\int_{-n}^{n} (3x^2 + 2x - 1)\,dx$

(A) $2n^2 + 2n$ (B) $2n^3 - 2n^2$ (C) $2n$ (D) $2n^3 - 2n$ (E) $2n^3$

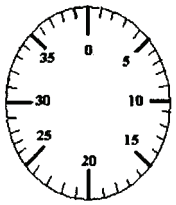
23. Slick Willy and Fingers Dolly each have a standard deck of cards. After shuffling several times each one flips over a randomly selected card. Determine the probability that at least one card is an ace. (nearest percent)

(A) 23 % (B) 15 % (C) 8 % (D) 4 % (E) 1 %

24. Karen E. Smith is currently a mathematics professor at the University of Michigan. She is highly recognized for her thesis and outstanding work in:

(A) commutative algebra (B) Fibonacci sequences (C) 4-dimensional hyper solids
(D) symmetry of triangles (E) logarithms

25. **The combination padlock shown below can be opened by turning right two or more whole turns and stopping at P. Then turn left one whole turn past P and stop at Q. Then turn right and stop at R. How many distinct combinations exist if P is a multiple of 5, Q is a factor of 10, and R is a perfect square? Stopping at 0 is not allowed.**



(A) 17 (B) 192 (C) 168 (D) 105 (E) 70

26. **The Mel Ting ice cream store offers 12 different flavors of ice cream. Mel makes banana splits using three scoops of ice cream. How many different three-scoop banana slips can he make?**

(A) 364 (B) 220 (C) 455 (D) 165 (E) 440

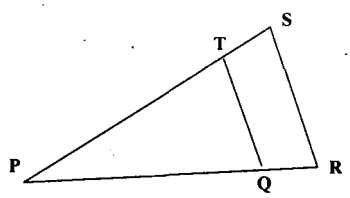
27. **If $x = \sqrt{64}$ and $\sqrt{64} = 8$, then $x = 8$ is an example of the _____ property of equality.**

(A) associative (B) commutative (C) distributive (D) identity (E) transitive

28. **$3333_4 - 222_3 + 11_2 =$ _____ 5**

(A) 1412 (B) 44442 (C) 1441 (D) 3122 (E) 4444

29. **In $\triangle PRS$, $QT \parallel RS$, $QR = x$, $PQ = 12$, $QT = 4$ and $RS = 3x$. Find x.**



(A) 2 (B) 1.5 (C) 3 (D) 4.5 (E) not enough information given

30. **In a plane, the locus of points equidistant from two given points is the _____ of the segment joining the points.**

(A) midpoint (B) perpendicular bisector (C) endpoint (D) length (E) angle bisector

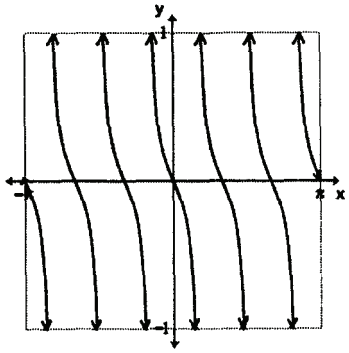
31. **Mr. White's high school math class has 30 students. 60% of the students are on his math team. 20 members of the class took a practice math test on Friday. 75% of those who took the math test were on the math team. What percentage of the class who were not on the math team took the practice math test?**

(A) $16\frac{2}{3}\%$ (B) $27\frac{7}{9}\%$ (C) $33\frac{1}{3}\%$ (D) $41\frac{2}{3}\%$ (E) 50%

32. The equation $3x^2 + 4x + k = 0$ has two imaginary roots. Which of the following is always a true statement about the value of k?

(A) $k < 0$ (B) $k < -2$ (C) $k > 2$ (D) $k > 1$ (E) $k > 0$

33. The equation $y =$ _____ will produce this graph.



(A) $-\frac{1}{3}\tan(4x + \pi)$ (B) $-\frac{1}{3}\cot(4x - \pi)$ (C) $\frac{1}{4}\cot(3x + \pi)$

(D) $\frac{1}{4}\tan(3x - \pi)$ (E) $-\frac{1}{4}\tan(3x + \pi)$

34. Simplify: $\sec(\frac{\pi}{2} - x) - \tan(\frac{\pi}{2} - x)\sin(\frac{\pi}{2} - x)$

(A) $-\cos(\frac{\pi}{2} - x)$ (B) $\sin x$ (C) $\sin 2x$ (D) $\sin(x - \frac{\pi}{2})$ (E) $\cos(\frac{\pi}{2} + x)$

35. The directrix of the parabola $y = x^2 - 6x + 5$ is:

(A) $x = 3$ (B) $y = -4$ (C) $y = -3\frac{3}{4}$ (D) $y = -4\frac{1}{4}$ (E) $x = -4\frac{3}{4}$

36. Find the determinant: $\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

37. A bag contains 3 yellow marbles, 4 green marbles, and 7 blue marbles. All of the marbles are the same size. Three marbles are randomly drawn without replacement. What is the probabilitythat the first one drawn is blue, second one is green, and the third one is blue? (nearest tenth)

(A) 7.7 % (B) 3.8 % (C) 6.1 % (D) 13.1 % (E) 6.4 %

38. Coach Mallery's math/science team consists of 3 girls and 5 boys. She wants to enter at least 2 student, but no more than 4 students in the math contest. In how many ways can she enter her students in the math contest?

(A) 162 (B) 154 (C) 112 (D) 98 (E) 66

39. Find the area (in square units) of the region bounded by $y = x^2 - 2x + 2$ and $y = 6 - x^2$.

(A) $9\frac{2}{3}$ (B) $11\frac{1}{3}$ (C) $10\frac{1}{3}$ (D) 12 (E) 9

40. Line l is a line tangent to the curve $y = x^3 - 5x + 1$ at point (x , y) and parallel to the x-axis. Which of the following could be the y-coordinate? (nearest tenth)

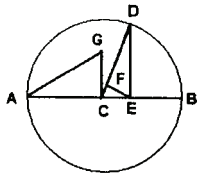
(A) −3.9 (B) 5.3 (C) −6.5 (D) 3.3 (E) 5.0

41. The integers greater than 0 are arranged in five columns as shown. If this pattern continues which column would contain the number 2009?

A	B	C	D	E
1	2	3	4	
	8	7	6	5
9	10	11	12	
	16	15	14	13

(A) A (B) B (C) C (D) D (E) E

42. Let AB be the diameter of the circle with center C with CG ⊥ AB, DE ⊥ AB, and EF ⊥ DC. If AE = 10, BE = 6.4, and CG = 1.8 then AG = ? (nearest tenth)



(A) 8.2 (B) 7.8 (C) 8.0 (D) 7.2 (E) 8.4

43. Find the mean of the median, mode, and range of 88, 72, 90, 85, 92, 67, & 85.

(A) $82\frac{1}{4}$ (B) 85 (C) 65 (D) 55 (E) $82\frac{5}{7}$

44. $x + 2$ is one of the factors of $6x^3 + 5x^2 - 17x - 6$. Which of the following is also a factor?

(A) $2x + 3$ (B) $6x - 1$ (C) $3x + 1$ (D) $2x - 1$ (E) $x - 3$

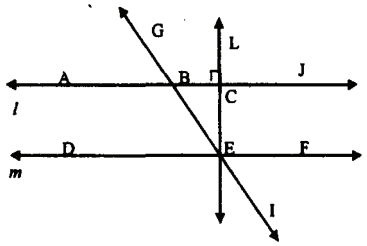
45. If $x - y = 5$ and $xy = 2$ then $x^3 - y^3 = ?$

(A) 155 (B) 399 (C) 95 (D) 250 (E) 133

46. If two-thirds of A equals the sum of three-fourths of B and one-half of C and B equals one-third of C, then C is what part of A?

(A) $\frac{1}{2}$ (B) $1\frac{1}{8}$ (C) $\frac{5}{6}$ (D) $\frac{8}{9}$ (E) $\frac{1}{4}$

47. The four lines in the figure are coplanar and $m//l$. Which of the following are true statements?



1. ∠CBE & ∠DEB are congruent
2. m∠FEI + m∠GBJ = m∠ABG + m∠DEI
3. m∠CBG = m∠FEI
4. ∠GED & ∠GEF are complementary angles

(A) 1 & 2 (B) 3 & 4 (C) 4 only (D) 2 & 3 (E) 1, 2, & 4

48. Willie Maykette deposits $100 in an account with an annual interest rate of 3% compounded monthly. Betty Dont deposits $100 in an account with an annual simple interest rate of 4.5%. The interest is added to their respective accounts at the end of each month. How much longer will it take Willie's original deposit to double than it will take Betty's to double? (nearest month)

(A) 5 months (B) 9 months (C) 11 months (D) 18 months (E) 23 months

49. The I. M. Dry ranch has a horse trough in the shape of a rectangular prism. It is 3 yards long, 1.5 feet wide and 14 inches high. How many whole gallons of water will it take to fill the trough without spilling over?

(A) 117 (B) 98 (C) 294 (D) 231 (E) 63

50. The 11^{th} Fibonacci number is 55 and the 13^{th} Fibonacci number is 144. Find the 12^{th} Lucas number?

(A) 89 (B) 123 (C) 144 (D) 199 (E) 322

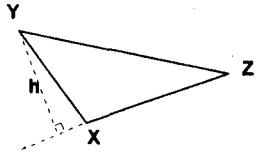
51. If $y^2 = 8 - 6i$ and $y^3 = 18 - 26i$ where $y = a + bi$ then $a + b$ equals:

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

52. If you start at $(-3\pi, 2)$ and travel horizontally 22 radians to the right, how many times will you cross the graph of $y = \sec(x)$?

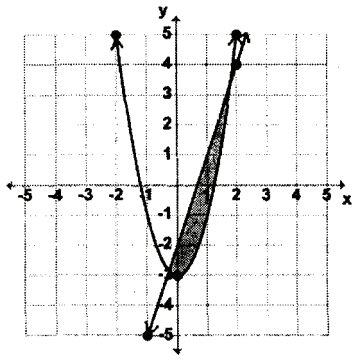
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

53. A triangle is drawn as shown. Find the height, h, if YZ = 15", m∠XZY = 30 °, and XZ = 10". (nearest tenth)



(A) 7.5 " (B) 8.7 " (C) 10.0 " (D) 12.5 " (E) 13.0 "

54. Which of the following system of inequalities would be best represented by the shaded region shown?



(A) $2x^2 + y \leq -3$, $3x + y \geq 2$

(B) $3x + y \geq -2$, $2x^2 - y \geq 3$

(C) $y \leq 2x^2 - 3$, $y \geq -3x - 2$

(D) $3x^2 - y \leq 2$, $2x - y \geq 3$

(E) $3x - y \geq 2$, $2x^2 - y \leq 3$

55. How many horizontal asymptotes does $f(x) = \frac{4x^2 - 5}{x - 2}$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

56. Find the angle of rotation, θ (closest approximation), where $0 < \theta < \frac{\pi}{2}$, such that the conic $2x^2 + 5x + 4y^2 - 3xy + 12 = 0$ contains no xy term in its equation.

(A) 0.294 (B) 0.340 (C) 0.409 (D) 0.491 (E) 1.080

57. Lon Kutter can mow the yard by himself in 1.5 hours. His sister, Ima, can mow the yard by herself in 2.5 hours. How long would it take to mow the yard if they both mowed the lawn together? (nearest minute)

(A) 64 minutes (B) 62 minutes (C) 60 minutes (D) 58 minutes (E) 56 minutes

58. Les Sense has a jar that contains Indian Head pennies and Lincoln pennies. There are less than 100 coins in the jar. The odds of selecting a Lincoln is $\frac{3}{5}$. If 10 more Lincolns are added to the jar, the probability of selecting a Lincoln becomes 50%. How Indian Heads are in the jar?

(A) 75 (B) 50 (C) 45 (D) 30 (E) 25

59. Find the harmonic mean of the roots of $x^3 - 3x^2 + 3.25x - 1.5 = 0$.

(A) 1 (B) $1\frac{1}{4}$ (C) $1\frac{11}{76}$ (D) $1\frac{5}{13}$ (E) $1\frac{39}{56}$

60. The three-dimensional vector $(-3, -2, 1)$ is perpendicular to which of the following vectors?

(A) $(-1, 2, 3)$ (B) $(-5, -1, -2)$ (C) $(3, 2, 1)$ (D) $(1, 1, 5)$ (E) $(3, -2, 1)$

University Interscholastic League
MATHEMATICS CONTEST
HS • District 2 • 2009
Answer Key

1. D	21. E	41. A
2. A	22. D	42. E
3. B	23. B	43. C
4. E	24. A	44. C
5. E	25. C	45. A
6. D	26. A	46. D
7. B	27. E	47. A
8. D	28. A	48. C
9. E	29. B	49. A
10. D	30. B	50. D
11. A	31. D	51. E
12. C	32. C	52. C
13. D	33. E	53. A
14. C	34. B	54. E
15. A	35. D	55. A
16. C	36. C	56. D
17. B	37. A	57. E
18. C	38. B	58. E
19. C	39. E	59. D
20. D	40. B	60. D