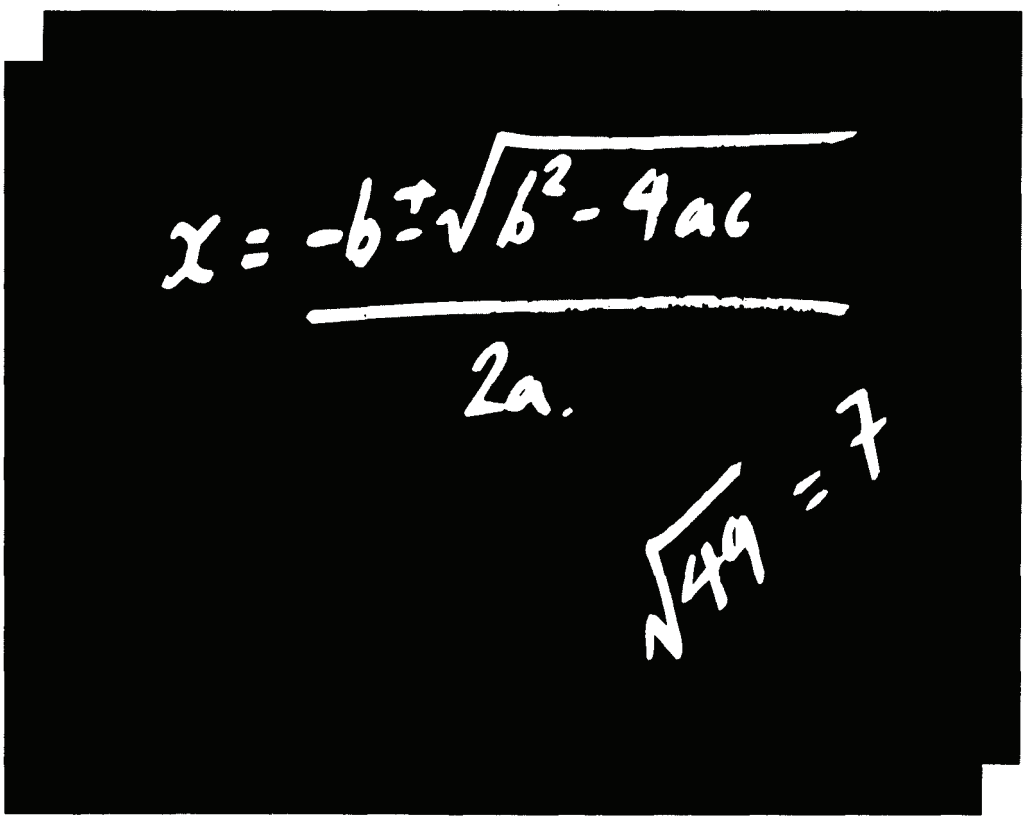
UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Invitational A • 2009



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $[1.2 \div (\frac{3}{5})^2 - (3)^{-1}] \times 4!$

(A) 88 (B) 72 (C) $48\frac{9}{25}$ (D) $27\frac{2}{3}$ (E) 8

2. 

The distances between the hash marks (|) are equal. Find P + R + S.

(A) -5.75 (B) $-\frac{1}{2}$ (C) $2\frac{1}{2}$ (D) 4.25 (E) 6

3. Phil Upp's truck gets 17 miles per gallon of gas. He has \$20.00 to spend on gas. If the cost of a gallon of is gas is \$3.50, how far can Phil drive? (nearest whole mile)

(A) 70 miles (B) 76 miles (C) 97 miles (D) 100 miles (E) 102 miles

4. Line *l* going through points $(-1, 3)$ and $(k, -5)$ is perpendicular to $x + 4y = 5$. Find k.

(A) -5 (B) -3 (C) -1 (D) 2 (E) 5

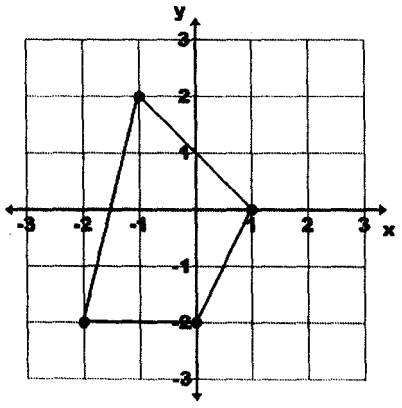
5. Simplify: $\left(\frac{6w^2+7w-3}{2w^3+5w^2+3w}\right)\left(\frac{w^2-w-2}{3w^2-7w+2}\right)$

(A) $\frac{w-3}{w^2-2}$ (B) $\frac{w+2}{3w^2+2w}$ (C) $\frac{1}{w}$ (D) $\frac{3w-2}{4+w}$ (E) $\frac{3w+1}{2w}$

6. Ima Whett paddles her kayak at a constant speed of 5 mph relative to the water. She paddles upstream for 1 hour 20 minutes. The return trip back only takes 1 hour 5 minutes. Which of the following is the closest approximation of the speed of the current?

(A) $\frac{1}{2}$ mph (B) $\frac{1}{3}$ mph (C) $\frac{5}{6}$ mph (D) $\frac{5}{8}$ mph (E) $\frac{1}{4}$ mph

7. Rene drew this quadrilateral on the coordinate plane below. The coordinates of the vertices are integers. What is the area of his quadrilateral?



(A) $6\frac{1}{2}$ units^2 (B) 7 units^2 (C) $7\frac{2}{3}$ units^2 (D) $8\frac{1}{4}$ units^2 (E) $8\frac{1}{2}$ units^2

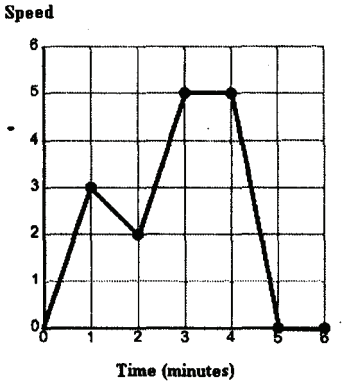
8. If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle then the line is _____ to the circle.

(A) complementary (B) diagonal (C) tangent (D) adjacent (E) secant

9. $\angle A$ and $\angle B$ are complementary . The ratio of $m\angle A$ to $m\angle B$ is 4:5. Find the ratio of $m\angle A$ to its supplement.

(A) 2:7 (B) 4:9 (C) 5:4 (D) 6:3 (E) 5:14

10. The graph best depicts Mei Strol's daily 6 minute walk. (speed is not truly linear in this case). During the time interval of 3 minutes to 4 minutes Mei is _____.

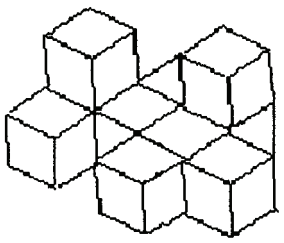


(A) walking on flat ground (B) walking at a constant speed (C) standing still
(D) decreasing speed (E) increasing speed

11. A line perpendicular to the axis of symmetry of a parabola is called the _____.

(A) focus (B) eccentricity (C) directrix (D) centroid (E) asymptote

12. The length of the sides of each of the small cubes is 1 cm. How many of the small cubes would need to be added to this figure to make a rectangular prism that is 4 cm long, 3 cm wide, and 2 cm tall?



(A) 14 (B) 13 (C) 12 (D) 11 (E) 10

13. A laser beam from the top of a 30-ft building hits an object on the ground 100 ft from the base of the building. The angle of depression of the laser to the object is: (nearest second)

(A) 14° 24' 11" (B) 16° 6' 9" C) 16° 41' 57" (D) 17° 4' 6" (E) 17° 27' 27"

14. Find the largest value of θ if $6\cos^2\theta + \cos\theta = 2$ and $\pi \leq \theta \leq 2\pi$.

(A) $\frac{6\pi}{5}$ (B) $\frac{4\pi}{3}$ (C) $\frac{5\pi}{3}$ (D) $\frac{8\pi}{5}$ (E) $\frac{5\pi}{4}$

15. Simplify $\sin\theta \cot\theta \sec\theta - \cos^2\theta$.

(A) $\sin^2\theta$ (B) $\csc^2\theta$ (C) $\sin 2\theta$ (D) $\sec^2\theta$ (E) 1

16. Let $x^5 - x^4 - px^3 + qx^2 - x - 1 = 0$, where p, q > 0. According to Descartes' Rule of Signs, how many possible positive roots are there?

(A) 5, 3, or 1 (B) 3 or 1 (C) 4, 2, or 0 (D) 2 or 0 (E) 0

17. The directrix of the parabola $8y = x^2 - 4x + 12$ is:

(A) $x = 2$ (B) $y = 1$ (C) $y = \frac{1}{8}$ (D) $x = -\frac{1}{4}$ (E) $y = -1$

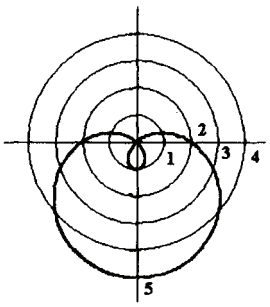
18. If 5 adults and 2 teenagers work together, they can do a job in 1 day. If only 2 adults work, then 6 teenagers must in order to do the job in 1 day. If no adults work and only 1 teenager works, how long will it take the teenager to do the job?

(A) $7\frac{1}{2}$ days (B) 8 days (C) $8\frac{1}{3}$ days (D) $8\frac{2}{3}$ days (E) 9 days

19. A function $y = f(x)$ is continuous on [a,b], if $f(a) < y_0 < f(b)$ then $y_0 = f(c)$ for some c in [a,b]. This theorem is the:

(A) Intermediate Value Theorem (B) Mean Value Theorem (C) Sandwich Theorem
(D) Max–Min Theorem (E) Fundamental Theorem of Calculus

20. Which of the following polar equations will produce this graph on the polar grid?



(A) $r = 1 + 5\sin\theta$ (B) $r = 2 - 4\sin\theta$ (C) $r = 1 + 2\cos\theta$

(D) $r = 2 - 3\sin\theta$ (E) $r = 2 - 5\cos\theta$

21. Let $f(x) = ax^5 - bx^4 - bx^3 + ax^2 + ax - b$. Find $f''(1)$.

(A) 22a — 18b (B) 18a — 6b (C) 22a — 19b (D) 18a — 6b (E) 22a — 7b

22. Seymore Endelite randomly selects two socks from his drawer to wear to school. The socks are identical except for their color and are not paired up. He has 8 blue socks, 6 black socks, and 4 white socks. What is the probability that he selects two black socks? (nearest percent)

(A) 9 % (B) 18 % (C) 32 % (D) 4 % (E) 10 %

23. Lotta Dough has a bag that contains one $100 bill, two $20 bills, three $10 bills, four $5 bills, and five $1 bills. The odds of her pulling out a $10 bill is 25%. How many $10 bills would have to be added to the bag to change the odds to 50%?

(A) 1 (B) 3 (C) 6 (D) 9 (E) 12

24. How many subsets containing 4 members can be made from the set {2, 1, 3, 4, 7, 11}?

(A) 6 (B) 10 (C) 15 (D) 20 (E) 21

25. Which of the following was the first Nigerian woman to be awarded a doctorate in mathematics?

(A) Emmy Noether (B) Freda Porter (C) Hypatia
(D) Karen E. Smith (E) Grace Alele Williams

26. Find the harmonic mean of the roots of $x^3 - 7x^2 + 14x - 8 = 0$.

(A) $1\frac{5}{7}$ (B) $1\frac{3}{4}$ (C) 2 (D) $2\frac{1}{3}$ (E) $2\frac{2}{5}$

27. If R, S, and T are distinct digits then $RST_2 - ST_3 - R_4$ has a numeric value in base 10 of:

(A) $2R - S + 2T$ (B) $-S$ (C) $S + T$ (D) $5R$ (E) $3R - S$

28. Find the ratio of the median to the mean of the following list of numbers.
2, 3, 5, 2, 4, 3, 2, 0, 5, 3, 5, 2

(A) 1:1 (B) 3:2 (C) 3:5 (D) 2:5 (E) 1:2

29. Missy Klas was absent the day of the algebra exam. She took the test the next day and made a 96. Her score raised the class average from 71 to 72. How many students, including Missy, took the test?

(A) 22 (B) 24 (C) 25 (D) 26 (E) 28

30. The set {..., −6, −4, −2, 0, 2, 4, 6, ...} is closed under which of the following operations :

I. addition II. subtraction III. multiplication IV. division

(A) all of these (B) I & III only (C) I, II, & III (D) II & IV only (E) none of these

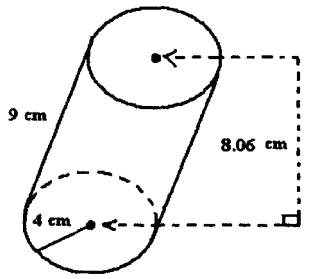
31. If the roots of $x^3 + bx^2 + cx + d = 0$ are -5, 1, and 3, then $b + c + d$ equals:

(A) -1 (B) 0 (C) 3 (D) 31 (E) 33

32. Mr. White's college math class has 40 students. 75% of the students are math majors. 32 of the students passed the final exam. 75% of those who passed the final exam are math majors. What percentage of the class who were not math majors passed the final exam?

(A) 8% (B) 20% (C) 25% (D) 75% (E) 80%

33. Find the lateral area, nearest square cm, of the oblique cylinder.



(A) 452 cm^2 (B) 352 cm^2 (C) 327 cm^2 (D) 226 cm^2 (E) 176 cm^2

34. If $a_1 = -4$, $a_3 = -9$, and $a_4 = 13.5$ are terms of a geometric sequence, then $a_2 =$ _____.

(A) -6 (B) -5 (C) 1.5 (D) 6 (E) 6.75

35. If $y^2 = -4 + 0i$ and $y^3 = 0 - 8i$ where $y = a + bi$ then $a + b$ equals:

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

36. Pop Eye takes his family sailing. They leave dock A and sail 1.5 miles on a course of 30° to buoy B. They turn and travel 1.75 miles on a bearing of 110° to buoy C. How far is it from buoy C to dock A? (nearest tenth)

(A) 1.6 miles (B) 2.0 miles (C) 2.3 miles (D) 2.5 miles (E) 2.7 miles

37. How many points of intersection occur when $r = 2\cos\theta + 1$ and $\theta = \pi$ are graphed on a polar coordinate system?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

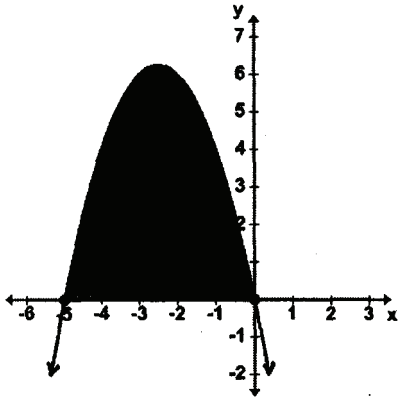
38. $\sum_{k=0}^{2} \left(kx + (k+1)y \right) = ?$

(A) $4x + 3y$ (B) $2x + 3y$ (C) $3y$ (D) $3x + 6y$ (E) $2x + y$

39. $F_0 = 0$ and $F_1 = 1$ are the first two Fibonacci numbers. Find F_{10}.

(A) 34 (B) 47 (C) 55 (D) 76 (E) 89

40. Find the area of the shaded region in square units.



(A) $18\frac{3}{4}$ (B) $19\frac{2}{3}$ (C) $20\frac{1}{2}$ (D) $20\frac{5}{6}$ (E) $21\frac{1}{4}$

41. Use the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, to transform the conic $x^2 + xy + y^2 = 3$ into an equation that does not contain an xy term. The equation is:

(A) $x^2 + y^2 = 9$ (B) $3x^2 + y^2 = 6$ (C) $x^2 - 2y^2 = 3$

(D) $x^2 + 3y^2 = 6$ (E) $3x^2 + y^2 = 3$

42. If $f(x) = \frac{2x+3}{4x-5}$, then $f'(1) =$ _____

(A) -22 (B) -5 (C) -2 (D) $\frac{1}{2}$ (E) 2

43. Betty Luzes rolls a fair die 4 times. What is the mathematical expectation of the sum of the outcomes of the 4 rolls ?

(A) 7 (B) 10.5 (C) 14 (D) 21 (E) 28

44. Five married couples attend the square dance planning meeting. How many committees of four people can be chosen if no committee is to include a husband-and-wife pair?

(A) 20 (B) 25 (C) 50 (D) 80 (E) 105

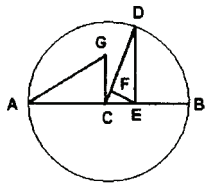
45. Let $R = \{1, 3, 5\}$, $S = \{0, 2, 4\}$, and $T = \{1, 2, 3\}$. How many elements are in $(R \cup T) \cap (S \cup T)$?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

46. The circles $x^2 + y^2 + 3x - 6y + 5 = 0$ and $2x^2 + 2y^2 + 5x - 6y + 3 = 0$ intersect in two points. The slope of the line through the two points of intersection is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{7}{6}$ (D) $\frac{3}{5}$ (E) $\frac{5}{6}$

47. Let AB be the diameter of the circle with center C with CG $\perp$ AB, DE $\perp$ AB, and EF $\perp$ DC. If AE = 9 and BE = 4 then DF = ?



(A) 5 (B) $5\frac{7}{13}$ (C) 6 (D) $6\frac{1}{2}$ (E) 7

48. How many of the following numbers are NOT solutions to $7 - 5|3x + 1| \geq -1$?

-0.987 $-0.777...$ $.222...$ 0.3 .12

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

49. It is precisely 2:45 pm on a circular clock. What is the measure of the smaller angle formed by the minute hand and the hour hand of the clock?

(A) 192° (B) 187.5 (C) 150° (D) 168° (E) 172.5°

50. $y^2 - x^2 = 0$ is an equation of a degenerate conic. Which of the following is the best graphical representation of this equation?

(A) point (B) line (C) parallel lines (D) intersecting lines (E) no graph

51. Let $f(x) = 4 - x$ and $g(x) = 3x - 5$ and $h(x) = 2x$. Find $h\Big(f\big(g(0)\big)\Big)$.

(A) 7 (B) 9 (C) 14 (D) 18 (E) 19

52. Points A, B, C, and D are the vertices of a square. Point E is on the interior of the square such that points A, B, and E form an equilateral triangle. A line segment connects points D and E. Another line segment connects points C and E. Find m$\angle$CED.

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{12}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{9}$ (E) $\frac{5\pi}{6}$

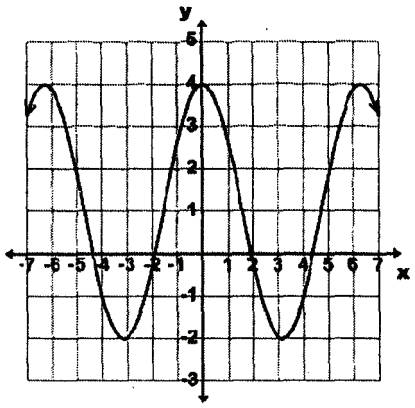
53. A regular deck of 52 cards is shuffled and the top five cards are dealt face up. What is the probability, nearest $\frac{1}{1000}$%, that all 5 cards are face cards (Jacks, Queens, Kings)?

(A) $\frac{1}{40}$% (B) $\frac{3}{100}$% (C) $\frac{2}{25}$% (D) $\frac{13}{200}$% (E) $\frac{23}{500}$%

54. Vector $v = (2, 9)$ is perpendicular to vector $w = (4, k)$. Find k.

(A) $-\frac{1}{9}$ (B) $-\frac{1}{8}$ (C) $-\frac{8}{9}$ (D) $-1\frac{1}{8}$ (E) -18

55. The graph shown is the graph of which of the following equations.



(A) $y = 1 + 3\sin(x)$ (B) $y = 3\cos(x + 1)$ (C) $y = \cos(3x + 1)$

(D) $y = 1 + 3\cos(x)$ (E) $y = 3\sin(x + 1)$

56. Point P has polar coordinates of $(4, \frac{2\pi}{3})$ and rectangular coordinates of (x , y). Where does point P lie on the Cartesian coordinate plane?

(A) QII (B) QIII (C) QIV (D) x-axis (E) y-axis

57. How many asymptotes does $f(x) = \frac{2 - 3x^2}{x - 1}$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

58. The slope of the line tangent to the curve $y = 2x^3 - 3x^2 - 5$ at $x = 2$ is 12. The point of intersection of the tangent line and the curve is:

(A) $(2, 2)$ (B) $(-1, -2)$ (C) $(-1, 2)$ (D) $(2, 1)$ (E) $(2, -1)$

59. Evaluate: $\int_{-n}^{n} (x^3 - 3x^2 - 5)\, dx$

(A) $\frac{n^4}{2}$ (B) $-10n$ (C) $-2n(n^2 + 5)$ (D) $\frac{n^4}{2} + 2n^2$ (E) $2n(n^2 - 5)$

60. The coordinates of the vertices of $\triangle ABC$ are $(-1, 2)$, $(1, 0)$ and $(-2, -2)$. The medians of the $\triangle ABC$ intersect at (x, y). Find $x + y$.

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{3}$ (D) $-\frac{2}{3}$ (E) -1

University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation A • 2009
Answer Key

1. B
2. A
3. C
4. B
5. C
6. A
7. B
8. C
9. A
10. B
11. C
12. A
13. C
14. C
15. A
16. B
17. E
18. D
19. A
20. D
21. A
22. E
23. B
24. C
25. E
26. A
27. E
28. A
29. C
30. C
31. A
32. E
33. D
34. D
35. E
36. D
37. B
38. D
39. C
40. D
41. B
42. A
43. C
44. D
45. D
46. A
47. B
48. C
49. E
50. D
51. D
52. E
53. B
54. C
55. D
56. A
57. C
58. E
59. C
60. D