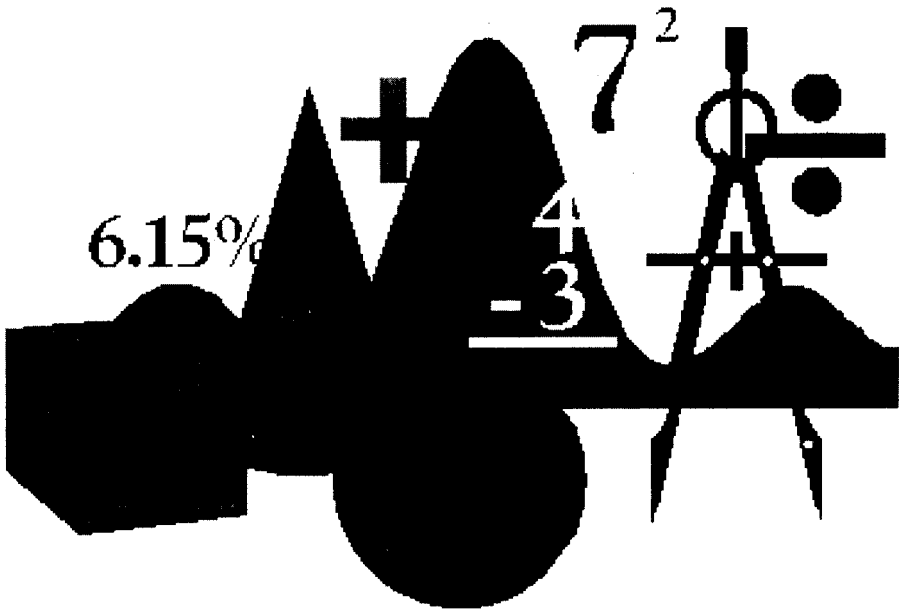
UNIVERSITY INTERSCHOLASTIC LEAGUE

Making a World of Difference

Mathematics

Regional • 2008



WRITE ALL ANSWERS WITH
CAPITAL LETTERS

DO NOT TURN THIS PAGE UNTIL
YOU ARE INSTRUCTED TO DO SO!

1. Evaluate: $8! \div 6! + (4)^2 \times 2! - 4 \div (6)^{-2} + 8$

(A)144 (B) 108 (C) -48 (D) $-79\frac{8}{9}$ (E) $-95\frac{8}{9}$

2. The universal set U = {r,e,g,i,o,n,a,l}. Subset R = {r,e,g,i,o,n} and subset G = {g,r,a,i,n}. How many elements are in the complement set of $R \cap G$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

3. Simplify: $\left[\left(a^{-5}\right)\left(b^{4}\right) \div \left(a^{4}\right)\left(b^{-5}\right) \div \left(a^{-3}b^{3}\right)\right]^{-1}$

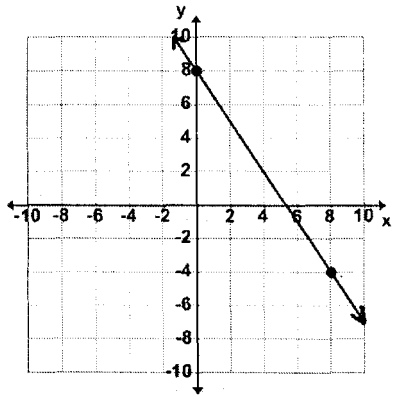
(A) $(a)^6(b)^{-6}$ (B) $(a)^6(b)^{-2}$ (C) $(a)^6(b)^4$ (D) $(a)^4(b)^{-2}$ (E) $(a)^{-4}(b)^{-6}$

4. The set $\{-1, 0, 1\}$ is closed under which of the following operations?

I. addition II. subtraction III. multiplication IV. division

(A) I, II, III & IV (B) I & III (C) I & II (D) III & IV (E) III only

5. Find the equation of the line containing the point (4, 2) that is perpendicular to the line shown .



(A) $3x - 2y = -2$ (B) $2x + 3y = 14$ (C) $2x - 3y = -14$

(D) $3x + 2y = 16$ (E) $2x - 3y = 2$

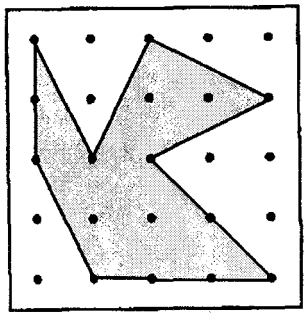
6. If $x - y = -6$ and $xy = 6$ then $x^3 - y^3 = ?$

(A) 216 (B) 57 (C) -18 (D) -108 (E) -324

7. Betty Whens is playing a game with a fair die. If she rolls a prime number she scores ten times the number rolled. If she rolls a composite number she only scores five times the number rolled. If she rolls anything else she scores the number rolled. What score should Betty expect on average?

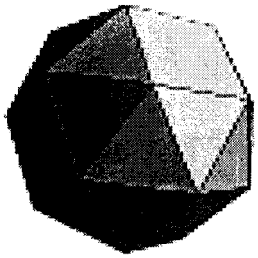
(A) $25\frac{1}{2}$ (B) $25\frac{1}{6}$ (C) $24\frac{5}{6}$ (D) $24\frac{2}{3}$ (E) $23\frac{1}{2}$

8. **Adjacent dots on the grid are 1 cm apart when measured vertically and horizontally. Find the area of the shaded figure shown.**



(A) 75 cm^2 (B) 8 cm^2 (C) 8.5 cm^2 (D) 9 cm^2 (E) 9.5 cm^2

9. **A snub cube is an Archimedian solid whose faces are triangles and squares. Six of the faces are squares. It has 24 vertices and 60 edges. How many of the faces are triangles?**



(A) 18 (B) 24 (C) 28 (D) 30 (E) 32

10. **The roots of the cubic $x^3 + Bx^2 + Cx + D = 0$ are $-1\frac{1}{2}, \frac{3}{4}$, and 3. Find $B + C + D$.**

(A) $-1\frac{5}{8}$ (B) $-2\frac{1}{4}$ (C) $-3\frac{3}{8}$ (D) $-4\frac{1}{2}$ (E) $-5\frac{1}{2}$

11. **Ulickum Stamp service is ordering special stamps to sell to their customers. They order three times as many 5¢ stamps as 10¢ stamps and twice the number of 25¢ stamps as 5¢ stamps. The total cost of the order is $87.50. How many stamps did they order?**

(A) 185 (B) 275 (C) 415 (D) 500 (E) 625

12. **If $y^2 = 3 + 4i$ and $y^3 = 2 + 11i$ where $y = a + bi$ then $a + b$ equals:**

(A) 12 (B) 11 (C) 8 (D) 5 (E) 3

13. **Flight 123 flies 789 km from Sumwear Airport to Ennywear Airport on a bearing of 160°. Flight 456 flies 789 km from Sumwear Airport to Nowear Airport on a bearing of 290°. What bearing will Flight 456 have to fly from Nowear directly to Ennywear ?**

(A) 95° (B) 110° (C) 115° (D) 135° (E) 145°

14. Which of the following expressions does not equal 1?

(A) $\tan(x)\cot(x)$ (B) $\frac{\cos(x)\csc(x)}{\cot(x)}$ (C) $\sin(x)\cot(x)\sec(x)$

(D) $\cos(x)\sec(x) - \tan(x)\cot(x)$ (E) $\sin^2(x) + \sin^2(x)\cot^2(x)$

15. Determine the frequency of $y = 2 - 3\cos\left[\frac{2}{3}\left(\frac{\pi}{2}x - \frac{3}{2}\right)\right]$.

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $1\frac{1}{2}$

16. Find the sum of a, b, and c from the geometric sequence, $2\frac{1}{4}$, a, b, $-\frac{2}{3}$, c, ...

(A) $-2\frac{1}{18}$ (B) $-\frac{1}{18}$ (C) $\frac{8}{9}$ (D) $\frac{17}{18}$ (E) $2\frac{17}{18}$

17. The vertical asymptote and the oblique asymptote of $f(x) = \frac{x^2 - x - 2}{x + 2}$ intersect at point (x , y). Find the value of y.

(A) 4 (B) 3 (C) -1 (D) -2 (E) -5

18. Simplify to the form $a + bi$: $\left(4 - \sqrt{-75}\right) \div \left(3 + \sqrt{-27}\right)$

(A) $-\frac{11}{12} - \frac{3\sqrt{3}}{4}i$ (B) $-\frac{11}{3} - 3\sqrt{3}\,i$ (C) $3 - \frac{2\sqrt{3}}{3}i$

(D) $-\frac{11}{12} - \frac{\sqrt{3}}{12}i$ (E) $\frac{19}{6} - \frac{3\sqrt{3}}{2}i$

19. Evaluate: $\int_{-a}^{a} 3x^2(1 - 2x)\,dx$

(A) $2a^3$ (B) $3a^4$ (C) $-3a^4$ (D) $a^3 - 1.5a^4$ (E) does not exist

20. Find an equation whose graph is a parabola having a focus point of (4,2) and a directrix of $y = 0$.

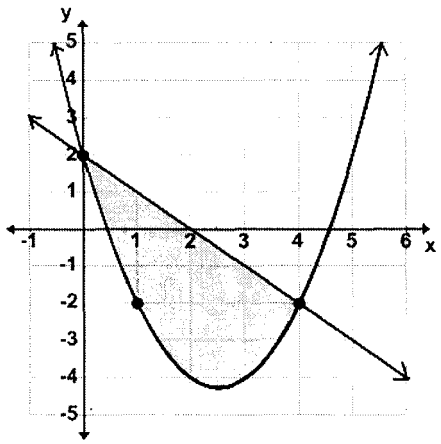
(A) $y^2 - 2y - 4x + 13 = 0$ (B) $x^2 - 4y + 12 = 0$ (C) $x^2 - 4x - 2y + 16 = 0$

(D) $y^2 - 4y - 8x + 12 = 0$ (E) $x^2 - 8x - 4y + 20 = 0$

21. Betty Canduit is trying to make obtuse triangles. She has 5 line segments whose measures are 3", 4", 5", 8", and 9". How many different obtuse triangles can she make?

(A) 1 (B) 3 (C) 4 (D) 6 (E) 8

22. **Find the area of the shaded region in square units.**

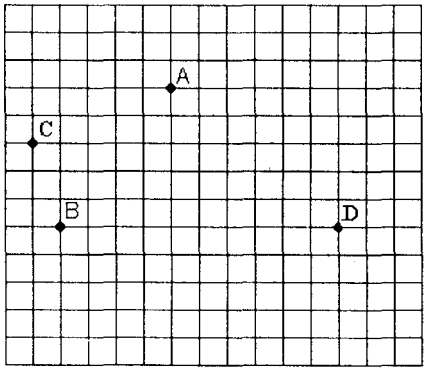


(A) $11\frac{1}{2}$ (B) $10\frac{2}{3}$ (C) 10 (D) $9\frac{3}{4}$ (E) $9\frac{1}{3}$

23. **How many different letter arrangements can be made by rearranging the letters in the word 'ENGINEER'?**

(A) 1440 (B) 2688 (C) 3360 (D) 5376 (E) 6720

24. **Line BD is parallel to the x-axis and perpendicular to the y-axis. If point A's coordinates are (1 , y) and point C's coordinates are (x , 1), then point D's coordinates are:**



(A) (2, — 7) (B) (— 3, — 2) (C) (— 3, 6) (D) (7, — 2) (E) (6, — 3)

25. **Which of the following vectors is neither parallel nor perpendicular to (4 , — 3)?**

(A) (8 , — 6) (B) (3 , 4) (C) (— 12 , 9) (D) (8 , 6) (E) (— 3 , — 4)

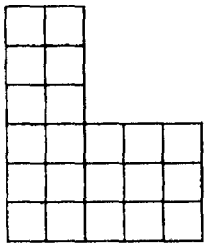
26. $88_9 - 444_5 + 2222_3 =$ _____ $_6$

(A) 322 (B) 120 (C) 100 (D) 212 (E) 324

27. **Find the 11^{th} term of the sequence 2, 15, 28, 255, 126, 1295, 344, ...**

(A) 730 (B) 1332 (C) 1727 (D) 9999 (E) 20735

28. 1 cm by 1 cm squares are stacked on top of each other to form the following figure. What is the total number of squares of any size are there in the figure?



(A) 37 (B) 36 (C) 35 (D) 34 (E) 33

29. Seymour Kalories has a box of chocolate candy eggs. He eats 3 of them before deciding to share the rest. He gives $\frac{1}{3}$ of what is left to his girl friend. He gives $66\frac{2}{3}\%$ of the remaining ones to his brothers and sisters. He gave the remaining 2 to his nice neighbors next door. How many chocolate eggs did have in his box originally?

(A) 9 (B) 12 (C) 15 (D) 18 (E) 24

30. The point (3, $-$ 4) is rotated 75 degrees counterclockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

(A) (5.0, 2.0) (B) (4.6, 1.9) (C) (4.0, 1.5) (D) (3.6, 1.4) (E) (3.0, 1.9)

31. If $\frac{3x+2}{3x-2} + \frac{3x-2}{3x+2}$ is written as the mixed number $A\frac{B}{C}$, then B is?

(A) 16 (B) 12 (C) 9 (D) 6 (E) 4

32. How many ordered pairs (x , y) are solutions to the equation $5x + 3y = 60$, where x and y are non-negative integers?

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

33. A triangle with side lengths 11 cm, 60 cm, and 61 is circumscribed around a circle. Find the radius of the circle. (nearest tenth)

(A) 30.5 cm (B) 27.7 cm (C) 15.0 cm (D) 5.5 cm (E) 5.0 cm

34. A right cylinder water tank is 8 feet high and has an inside diameter of 12 feet. The tank contains 5000 gallons. What percent of the tank's capacity contains water? (neatest percent)

(A) 45% (B) 59% (C) 68% (D) 74% (E) 90%

35. Let $f(x) = \sqrt{9 - x^2}$ Which of the following statements is a false statement.

(A) f is not continuous at 3
(B) the right hand derivative at 3 exists
(C) the left hand derivative at 3 exists
(D) f is not differentiable at 3
(E) $f(3) = f(-3)$

36. **Millie Ton weighs 105 pounds and is sitting on a seesaw 18 feet from the middle. Les Pounds weights 140 pounds and is sitting on the opposite end of the seesaw. How far from the center must Les sit to balance the seesaw?**

(A) 13 ft. 6 in. (B) 14 ft. $\frac{3}{4}$ in. (C) 14 ft. $7\frac{1}{2}$ in. (D) 15 ft. $2\frac{1}{4}$ in. (E) 15 ft. 9 in.

37. **Simplify:** $\ln\left(\frac{x}{x-1}\right) - \ln(x^2 - 1) + \ln\left(\frac{x+1}{x}\right)$

(A) $\ln(x+1)^2$ (B) $\ln(x-1)^2$ (C) $-2\ln(x-1)$ (D) $2\ln(x^2-1)$ (E) $-2\ln(x^2+1)$

38. **If** $a_1 = -2$, $a_2 = 0$, $a_3 = 2$ **and** $a_n = (a_{n-1})(a_{n-3}) - (a_{n-2})$, **where** $n \geq 4$, **then** a_{11} **is:**

(A) -4 (B) -2 (C) 0 (D) 2 (E) undefined

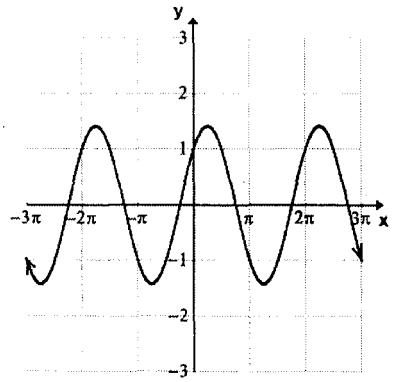
39. **How many points of intersection occur when** $r = \cos(2\theta)$ **and** $\theta = 0$ **are graphed on a polar coordinate system?**

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

40. **Simplify:** $\frac{\sin\theta}{\csc\theta - \cot\theta}$

(A) $\sec\theta - 1$ (B) $\frac{1}{\cos\theta}$ (C) $\tan\theta$ (D) $1 + \cos\theta$ (E) $\sec\theta + \cos\theta$

41. **The graph shown is the graph of which of the following equations.**



(A) $y = \sin(x)\cos(x)$ (B) $y = \sin(x) - \cos(x)$ (C) $y = \cos^2(x) - \sin^2(x)$

(D) $y = \sin^2(x) - \cos^2(x)$ (E) $y = \sin(x) + \cos(x)$

42. **Moe Speed is driving from his apartment in the city to his parent's farm. A third of the distance is on city streets, a third on suburb roads, and a third on rural highways. He travels at a constant speed of 25 mph on city streets, 35 mph on suburb roads, and 55 mph on rural highways. What was Moe's average speed for the trip? (nearest tenth)**

(A) 38.3 mph (B) 36.4 mph (C) 34.6 mph (D) 29.0 mph (E) 27.5 mph

43. Three white marbles and a blue marble are placed in a brown bag. The marbles are the same size and shape. Abe reaches in the bag and gets a marble, then Ben, then Cal, and finally Don. What is the probability that Cal gets the blue marble?

(A) 25 % (B) $33\frac{1}{3}$ % (C) 50 % (D) $66\frac{2}{3}$ % (E) 75 %

44. A square and an equilateral triangle have the same area. The perimeter of the square is 24 cm. Find the perimeter of the equilateral triangle. (nearest tenth)

(A) 23.7 cm (B) 27.4 cm (C) 29.7 cm (D) 31.2 cm (E) 36.5 cm

45. Find the angle of rotation, θ (nearest degree), where $0° < \theta < 90°$, such that the conic $3x^2 + 4\sqrt{3}\,xy - y^2 = 7$ contains no xy term in its equation.

(A) 25 ° (B) 27 ° (C) 30 ° (D) 33 ° (E) 35 °

46. Let $f(x) = \frac{1 + 2x - 3x^2}{4x + 5}$. Find $f'(-1)$.

(A) 24 (B) 16 (C) 0 (D) -4 (E) -8

47. Which of the following is a solution to $|3x - 7| - 5 < 9$?

(A) 3.333 (B) 7.777 (C) 9.999 (D) -5.555 (E) -3.759

48. Ten girls and 7 boys signed up to participate on an academic team. Each team consists of 5 members. How many different teams can be formed if at least 3 of the members must be girls?

(A) 1927 (B) 2520 (C) 3308 (D) 4095 (E) 4242

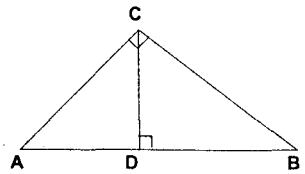
49. The arithmetic mean of two positive numbers, x and y, is $13\frac{1}{2}$. The harmonic mean of x and y is $13\frac{1}{3}$. Find the geometric mean of x and y.

(A) $13\frac{1}{12}$ (B) $5\sqrt{6}$ (C) $13\frac{7}{12}$ (D) $6\sqrt{5}$ (E) $13\frac{5}{6}$

50. In the expansion of $(x - 2y)^7$, the sum of the coefficients of the 3rd and the 6th term is:

(A) -756 (B) -644 (C) -588 (D) -512 (E) -504

51. Find AC if AD = 40 cm. and BD = 80 cm. (nearest tenth)



(A) 41.4 cm (B) 56.6 cm (C) 62.9 cm (D) 69.3 cm (E) 98.0 cm

52. Which of the following mathematicians created a set of rods (originally made of ivory) that were used for calculating products and quotients and for extracting square roots?

(A) Rene Descartes (B) Diophantus (C) Euclid (D) Sophie Germain (E) John Napier

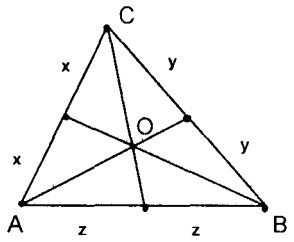
53. Forty-four students at Hy Tek High have iPods. Sixty-six Hy Tek students have cell phones. Thirty of the Hy Tek students have both an iPod and a cell phone. How many students at Hy Tek High have at least an iPod or a cell phone?

(A) 110 (B) 80 (C) 52 (D) 36 (E) 14

54. R_1, R_2 and R_3 are the roots of the equation $6x^3 + 11x^2 - 14x = 24$. R_2 and R_3 are the roots of the equation $2x^2 + x = 6$ as well. Find R_1.

(A) $1\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $-\frac{2}{3}$ (D) $-1\frac{1}{3}$ (E) -2

55. The point of intersection, O, of the triangle is called the _____.



(A) center (B) centroid (C) circumcenter (D) incenter (E) orthocenter

56. If $f(x) = \frac{3x+1}{2}$ and $g(x) = \frac{x-2}{3}$, then $g^{-1}[f^{-1}(0)]$ equals:

(A) -1 (B) $-\frac{1}{2}$ (C) $-\frac{1}{3}$ (D) 1 (E) 2

57. Determine the type of degenerate conic this equation gives: $3x^2 + 3y^2 + 6 = 6$.

(A) point (B) 1 line (C) 2 parallel lines (D) 2 intersecting lines (E) no graph

58. If $f''(x) = 16$ and $f'(\frac{1}{2}) = 3$ and $f(\frac{1}{2}) = \frac{3}{2}$, then $f(\frac{3}{2}) =$ _____.

(A) $\frac{25}{2}$ (B) $\frac{17}{2}$ (C) $\frac{15}{2}$ (D) $\frac{3}{2}$ (E) 8

59. P, Q, and R are positive integers. If $P + \frac{1}{Q + \frac{1}{R}} = \frac{42}{17}$, then find $P + Q + R$.

(A) 23 (B) 12 (C) 10 (D) 8 (E) 4

60. Tuff Teef has a dozen identical jawbreakers. How many ways can he put them into 6 bags so that each bag has at least 1 jawbreaker in it?

(A) 462 (B) 342 (C) 330 (D) 288 (E) 120

University Interscholastic League
MATHEMATICS CONTEST
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Answer Key

1. C	**21. B**	**41. E**
2. C	**22. B**	**42. C**
3. C	**23. C**	**43. A**
4. E	**24. D**	**44. B**
5. E	**25. D**	**45. C**
6. E	**26. C**	**46. A**
7. B	**27. B**	**47. A**
8. D	**28. C**	**48. E**
9. E	**29. B**	**49. D**
10. B	**30. B**	**50. C**
11. D	**31. A**	**51. D**
12. E	**32. A**	**52. E**
13. D	**33. E**	**53. B**
14. D	**34. D**	**54. D**
15. A	**35. B**	**55. B**
16. B	**36. A**	**56. D**
17. E	**37. C**	**57. A**
18. A	**38. D**	**58. A**
19. A	**39. D**	**59. B**
20. E	**40. D**	**60. A**